inkPhysics

ANSWER BOOK

Resonant circuits and filters

Electronics · Year 13

AQA 3.13.2

17 questions · 51 marks

NAVY EDITION

SECTION A · Warm-up

A1 An inductor's reactance $2\pi fL$ rises with frequency; a capacitor's reactance $1/2\pi fC$ falls (1). At resonance the two are equal in size and opposite in sense, so they cancel and only the circuit's resistance is left to limit the current, which is therefore a maximum (1). Saying the reactances 'become zero' is the standard error: each is still large, they simply cancel. [2]

A2 $Q = f_0/f_B$, the resonant frequency divided by the bandwidth, the bandwidth being the width of the peak between the two frequencies at which the current has fallen to $1/\sqrt{2}$ of its peak value (1). It is a **pure** number with no unit (1). Quoting a Q factor as a percentage is the standard error, and it earns nothing: a Q of 10 is not 10%, it is a peak ten times narrower than its own centre frequency. [2]

A3 Band-pass: take the output **across** the resistor, because the current, and so the pd across R, is largest at resonance (1). Band-stop: take the output **across** the series inductor and capacitor, whose combined reactance vanishes at resonance, so the output there falls to zero while everything far from resonance gets through (1). Naming the component without saying why is the usual half-answer; the examiner wants the output terminals named. [2]

A4 Output **across** the capacitor gives a **low-pass** filter, since the capacitor's reactance is large at low frequency and small at high, so high frequencies are shorted away (1). Output **across** the resistor gives a **high-pass** filter (1). Swapping the two is the standard error; anchor it on the capacitor, which passes fast signals and blocks slow ones. [2]

A5 It is the frequency at which the capacitor's reactance equals the resistance, $f_c = 1/2\pi RC$ (1), and there the output has fallen to $1/\sqrt{2}$, about 0.71, of its full value, which is the half-power point (1). Describing the cut-off as the frequency where the output 'stops' is the standard error: the response slides away over a decade or more and there is no sharp edge. [2]

A6 The resonant frequency does not move at all, since f_0 depends on L and C alone (1). The peak is lower and broader, because Q has fallen and the bandwidth f_0/Q has widened (1). Claiming that resistance shifts the peak sideways is the standard error, and it costs the mark every time. [2]

SECTION B · Standard

B1 $LC = 1.0 \times 10^{-3} \times 100 \times 10^{-9} = 1.0 \times 10^{-10}$ (1), whose square root is 1.0×10^{-5} s (1). $f_0 = 1/(2\pi \times 1.0 \times 10^{-5}) = 1.59 \times 10^4$ Hz, that is 15.9 kHz (1). Clear the millihenries and nanofarads first, multiply, then take the root: leaving the prefixes in, or taking the root before multiplying, is where almost every lost mark in this calculation goes. [3]

B2 $f_B = f_0/Q = 15.9/10 = 1.59$ kHz (1). The passband runs half a bandwidth either side of resonance (1), from 15.1 kHz to 16.7 kHz (1). Running the band a whole bandwidth either side of f_0, which doubles it, is the standard error: f_B is the total width, not the half-width. [3]

B3 $Q = 2\pi \times 15915 \times 1.0 \times 10^{-3}/25$ (1) $= 4.0$ (1). $f_B = f_0/Q = 15.9/4.0 = 4.0$ kHz (1). Compare with 10 Ω, which gives Q = 10 and a bandwidth of 1.59 kHz: more resistance means less Q and a broader peak, at exactly the same centre frequency. Writing Q with R on top is the standard error, and it makes resistance improve the circuit, which it never does. [3]

B4 $f_c = 1/(2\pi RC) = 1/(2\pi \times 1600 \times 0.10 \times 10^{-6})$ (1) $= 1/(2\pi \times 1.6 \times 10^{-4})$ (1) $= 995$ Hz, just under 1.0 kHz (1). Reading 0.10 μF as 0.10×10^{-9} F is the standard error here, and it puts the cut-off a thousand times too high. [3]

B5 $C = 1/(2\pi f_c R)$ (1) $= 1/(2\pi \times 2500 \times 4700) = 1.35 \times 10^{-8}$ F, that is 13.5 nF (1). The output is taken **across** the capacitor (1). Rearranging to $C = 2\pi f_c R$ is the standard slip: check the answer is a small fraction of a microfarad, because a capacitance of 7.4×10^7 F does not exist. [3]

B6 At 360 pF: $f_0 = 1/(2\pi\sqrt{(1.0 \times 10^{-3} \times 360 \times 10^{-12})}) = 2.65 \times 10^5$ Hz, that is 265 kHz (1). At 40 pF: 7.96×10^5 Hz, that is 796 kHz (1). The ratio is 3.0 to 1 (1). Note the square root at work: nine to one in capacitance is only three to one in frequency. Expecting the frequency to change by the full factor of nine is the standard error. [3]

B7 $f_B = 41.6 - 38.4 = 3.2$ kHz (1). $Q = f_0/f_B = 40.0/3.2 = 12.5$ (1), a pure number. Taking only the gap from the peak to one half-power point, 1.6 kHz, is the standard error, and it doubles the Q. [2]

SECTION C · Stretch

C1 Rearranging $f_0 = 1/(2\pi\sqrt{LC})$ gives $C = 1/(L(2\pi f_0)^2)$ (1) $= 1/(250 \times 10^{-6} \times (2\pi \times 2.00 \times 10^5)^2) = 2.53 \times 10^{-9}$ F, that is 2.53 nF (1). $Q = f_0/f_B = 200/8.0 = 25$ (1). Rearranging the Q expression, $R = 2\pi f_0 L/Q$ (1) $= 2\pi \times 2.00 \times 10^5 \times 250 \times 10^{-6}/25 = 12.6\ \Omega$ (1). Less resistance would push Q higher and the bandwidth below 8.0 kHz (1), so the circuit would start shaving the outer sidebands off the station it is tuned to and the received audio would lose its top frequencies (1). Two standard errors sink this design: leaving the 250 μH as 250 in the capacitance step, and quoting the Q of 25 as 25%. A Q factor is a bare number, and a bandwidth is never chosen narrower than the signal it must carry. [7]

C2 A **high-pass** filter, since the wanted signal lies above the interference (1). $C = 1/(2\pi f_c R) = 1/(2\pi \times 120 \times 22 \times 10^3) = 6.03 \times 10^{-8}$ F, about 60 nF (1). At 50 Hz, $f/f_c = 50/120 = 0.417$, so the output fraction is 0.38 (1): about 38% of the hum still gets through, while speech at 300 Hz passes at 0.93 (1). Assuming a filter removes everything below its cut-off is the standard error; a single RC stage only attenuates, and the roll-off is gentle. [4]

C3 The notch sits at the resonant frequency, so $L = 1/(C(2\pi f_0)^2)$ (1) $= 1/(1.0 \times 10^{-6} \times (2\pi \times 50)^2) = 10.1$ H (1). That is an enormous inductor, physically large, heavy and resistive, so a real 50 Hz notch is built round an op-amp instead (1). The output is taken **across** the series L and C, whose combined reactance vanishes at 50 Hz (1). The standard error is arithmetic: at mains frequency the inductance comes out in henries, not millihenries, and an answer of 10 mH means a factor of 10^3 has been dropped somewhere in the prefixes. [4]

C4 $f_B = f_0/Q = 909/300 = 3.03$ kHz (1). The bandwidth should be about the channel spacing, 9 kHz, which needs $Q = 909/9 = 101$ (1). At Q = 300 the passband is only 3.03 kHz wide, far narrower than the 9.0 kHz the station occupies (1), so the outer sidebands, which carry the highest audio frequencies, are cut away and the programme arrives dull and muffled (1). The standard error is to treat a higher Q as automatically better; selectivity is bought at the cost of bandwidth, and a real signal is a band of frequencies rather than a line. [4]