InkPhysics

ANSWER BOOK

Rotational motion and moment of inertia

Engineering physics · Year 13

AQA 3.11.1

17 questions · 51 marks

NAVY EDITION

SECTION A · Warm-up

A1 Angular velocity is the angle swept out per unit time about the axis, $\omega = \Delta\theta/\Delta t$ (1). Its unit is rad s^{-1} (1). Every point of a rigid body shares the same ω, however far from the axis it sits. [2]

A2 One revolution is 2π rad and one minute is 60 s, so $\omega = 2400 \times 2\pi/60$ (1) = 251 rad s^{-1} (1). Leaving the speed in rev min^{-1} is the standard error here, and it poisons every line that follows. [2]

A3 It measures the body's resistance to angular acceleration about that axis, the rotational counterpart of mass (1). It depends on how much mass the body has and on how that mass is distributed about the axis, since $I = \Sigma mr^2$ weights each particle by the square of its distance (1). [2]

A4 $I = \Sigma mr^2$, so each particle counts by the square of its distance from the axis (1). The hoop carries all its mass at the full radius, while the disc spreads its mass inward where r^2 is small, so the hoop's sum is larger, in fact twice as large (1). [2]

A5 Displacement s becomes angular displacement θ; velocity v becomes angular velocity ω; acceleration a becomes angular acceleration α (1); mass m becomes moment of inertia I (1). [2]

A6 $E_k = ½I\omega^2$ (1), the exact twin of $½mv^2$ with I standing in for m and ω for v. [1]

SECTION B · Standard

B1 Convert first: $\omega_2 = 900 \times 2\pi/60 = 94.2$ rad s^{-1} (1). $\alpha = (\omega_2 - \omega_1)/t$ (1) = 94.2/6.0 = 15.7 rad s^{-2} (1). Dividing 900 by 6.0 without converting gives 150, a number in the wrong units entirely. [3]

B2 $\omega = 45 \times 2\pi/60 = 4.71$ rad s^{-1} (1). $E_k = ½I\omega^2$ (1) = ½ × 0.045 × 4.71^2 = 0.50 J (1). The conversion is again the marked step; a raw 45 in the formula inflates the energy about ninety-fold. [3]

B3 Each mass sits at r = 0.80/2 = 0.40 m, not 0.80 m (1). $I = \Sigma mr^2 = 2 \times 0.60 \times 0.40^2$ = 0.192 kg m^2 (1). At 0.10 m, $I = 2 \times 0.60 \times 0.10^2$ = 0.012 kg m^2 (1), smaller by a factor of 16, because r^2 rewards distance so heavily. Using the rod's full length as r is the standard slip. [3]

B4 $\theta = 250 \times 2\pi = 1571$ rad (1). Use $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (1): $0 = 120^2 + 2\alpha \times 1571$, so $\alpha = -4.58$ rad s^{-2}, a deceleration of 4.58 rad s^{-2} (1). Revolutions must become radians before they enter an angular SUVAT equation. [3]

B5 Halve the diameter: $r = 0.90/2 = 0.45$ m (1). $v = \omega r = 12 \times 0.45 = 5.4$ m s^{-1} (1). $a = \alpha r = 3.0 \times 0.45 = 1.35$ m s^{-2} (1). Feeding the diameter straight in doubles both answers, and it is this option's most reliable lost mark. [3]

B6 $I = ½ \times 25 \times 0.30^2 = 1.125$ kg m^2 (1). $\omega = 2000 \times 2\pi/60 = 209$ rad s^{-1} (1). $E_k = ½I\omega^2 = ½ \times 1.125 \times 209^2 = 2.5 \times 10^4$ J (1). Both conversions, rev min^{-1} to rad s^{-1} and mass to I, must happen before the energy formula. [3]

B7 $\theta = \omega_1 t + \alpha t^2/2$ (1) $= 0 + 4.0 \times 8.0^2/2 = 128$ rad (1). Revolutions $= 128/2\pi = 20.4$ (1). Dividing by 2π is the return trip of the usual conversion, and forgetting it leaves an answer 6.3 times too large. [3]

SECTION C · Stretch

C1 The store is $E_k = ½I\omega^2$. Doubling ω quadruples the energy, because ω is squared (1), while doubling the mass only doubles I and so only doubles the energy (1). Placing mass at the rim raises I for the same mass, because $I = \Sigma mr^2$ counts each particle by the square of its distance from the axis (1), so a rim-loaded wheel stores more energy at the same spin rate than a solid one of equal mass (1). Real flywheel batteries are therefore modest rim-heavy discs spun very fast in a vacuum, not heavy ones spun slowly. [4]

C2 $I = ½ \times 180 \times 0.50^2 = 22.5$ kg m^2 (1). $\omega_1 = 3600 \times 2\pi/60 = 377$ rad s^{-1} and $\omega_2 = 1800 \times 2\pi/60 = 188$ rad s^{-1} (1). $E_1 = ½ \times 22.5 \times 377^2 = 1.60 \times 10^6$ J (1). $E_2 = ½ \times 22.5 \times 188^2 = 4.00 \times 10^5$ J (1). Usable energy $= 1.60 \times 10^6 - 4.00 \times 10^5 = 1.20 \times 10^6$ J (1). Time $= 1.20 \times 10^6/40\,000 = 30$ s (1). Note that halving the spin rate empties three quarters of the store, not half, because $E_k \propto \omega^2$: an answer of half the energy has forgotten the square. Both speeds must also be converted from rev min^{-1} first. [6]

C3 Disc: $I = ½ \times 60 \times 0.40^2 = 4.8$ kg m^2, so $E_k = ½ \times 4.8 \times 150^2 = 5.4 \times 10^4$ J (1). Ring: $I = 60 \times 0.40^2 = 9.6$ kg m^2, so $E_k = ½ \times 9.6 \times 150^2 = 1.1 \times 10^5$ J (1). The ring stores twice as much for the same mass and the same spin rate (1), so the ring is the better store: mass held at full radius counts fully in Σmr^2, whereas the disc's inner mass contributes little (1). [4]

C4 $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ gives $\alpha = (20^2 - 8.0^2)/(2 \times 42)$ (1) $= 4.0$ rad s^{-2} (1). Then $t = (\omega_2 - \omega_1)/\alpha = 12/4.0 = 3.0$ s (1). Energy: $\Delta E_k = ½I(\omega_2^2 - \omega_1^2)$ (1), so $1680 = ½ \times I \times (400 - 64)$ and $I = 10.0$ kg m^2 (1). The energy step must use the difference of the squares; using $½I(\Delta\omega)^2$ with $\Delta\omega = 12$ is the standard error and gives an I more than twice too large. [5]