InkPhysics

ANSWER BOOK

Torque, angular momentum and rotational power

Engineering physics · Year 13

AQA 3.11.1

17 questions · 51 marks

NAVY EDITION

SECTION A · Warm-up

A1 Torque is the turning effect of a force about the axis, the product of the force and its perpendicular distance from the axis, $T = Fr$ for a force applied tangentially at radius r (1). Its unit is N m (1). [2]

A2 $T = I\alpha$ (1), where T is the **net** torque in N m, I the moment of inertia about that axis in kg m^2 and α the angular acceleration in rad s^{-2} (1). It is the rotational form of $F = ma$, and it is the net torque, not the drive torque, that belongs in it. [2]

A3 If no resultant external torque acts on a system, its total angular momentum $I\omega$ stays constant (1). Internal forces, muscles, clutches or a collapsing structure, cannot change it, however much they rearrange the mass (1). [2]

A4 No external torque acts, so her angular momentum $I\omega$ is conserved (1). Pulling her arms in moves mass closer to the axis, reducing I, so ω must rise to keep the product constant (1). Nothing gives her an extra twist: she is not pushing against anything outside herself. [2]

A5 Its large moment of inertia means a given torque fluctuation produces only a tiny change in ω, since $\alpha = T/I$ (1). It absorbs angular momentum during each power stroke and returns it during the gaps between them, so the shaft turns almost uniformly (1). [2]

A6 kg m^2 s^{-1}, equivalently N m s (1), following straight from $I\omega$ with I in kg m^2 and ω in rad s^{-1}. [1]

SECTION B · Standard

B1 The radius is half the diameter: $r = 0.60/2 = 0.30$ m (1). $T = Fr = 250 \times 0.30 = 75$ N m (1). Using the diameter gives 150 N m, double the true torque, and it is the commonest slip in this calculation. [2]

B2 $\alpha = T/I$ (1) $= 20/8.0 = 2.5$ rad s^{-2} (1). $\omega_2 = \omega_1 + \alpha t = 0 + 2.5 \times 15 = 37.5$ rad s^{-1} (1). [3]

B3 $\omega = 2400 \times 2\pi/60 = 251$ rad s^{-1} (1). $P = T\omega$, so $T = P/\omega$ (1) $= 45\,000/251 = 179$ N m (1). $P = T\omega$ demands ω in rad s^{-1}; dividing by 2400 instead is the standard error and gives an answer about 26 times too small. [3]

B4 Angular momentum to remove: $\Delta(I\omega) = 0.75 \times 24 = 18$ kg m^2 s^{-1} (1). Angular impulse $T\Delta t = \Delta(I\omega)$ (1), so $\Delta t = 18/3.0 = 6.0$ s (1). Half the torque would take twice as long, delivering the same angular impulse. [3]

B5 $I\omega$ is conserved: $5.6 \times 1.5 = 2.0 \times f$ (1), so $f = 4.2$ rev s^{-1} (1). Energy ratio = $(2.0 \times 4.2^2)/(5.6 \times 1.5^2) = 2.8$ (1). Revolutions per second are safe here because only the ratio matters, but the energy rises even though the momentum did not: her muscles did the work of hauling her arms inward. [3]

B6 $W = T\theta$ (1) $= 12 \times 500 = 6000$ J (1). $P = W/t = 6000/25 = 240$ W (1). The same answer comes from $P = T\omega$ with $\omega = 500/25 = 20$ rad s^{-1}: $12 \times 20 = 240$ W. [3]

B7 Net torque = $30 - 6.0 = 24$ N m (1). $\alpha = T/I = 24/4.0 = 6.0$ rad s^{-2} (1). $\theta = \alpha t^2/2 = 6.0 \times 10^2/2 = 300$ rad (1). Revolutions = $300/2\pi = 48$ (1). $T = I\alpha$ takes the **net** torque; using 30 N m gives 7.5 rad s^{-2} and is the standard error here. [4]

SECTION C · Stretch

C1 No external torque acts on the pair, so $I\omega$ is conserved: $1.8 \times 120 = (1.8 + 1.2)\omega$ (1), giving $\omega = 72$ rad s^{-1} (1). E_k before = $½ \times 1.8 \times 120^2 = 12960$ J (1). E_k after = $½ \times 3.0 \times 72^2 = 7776$ J (1). Energy lost = $12960 - 7776 = 5184$ J, heating the slipping faces (1). For the time, apply the angular impulse to the disc that was at rest: $T\Delta t = \Delta(I\omega) = 1.2 \times 72 = 86.4$ kg m^2 s^{-1}, so $\Delta t = 86.4/9.0 = 9.6$ s (1). Angular momentum survives a coupling; kinetic energy never does, and answers that conserve energy here score nothing. [6]

C2 $\omega = 3000 \times 2\pi/60 = 314$ rad s^{-1} (1). Angular impulse: $T\Delta t = \Delta(I\omega) = 220 \times 314 = 6.91 \times 10^4$ kg m^2 s^{-1}, so $t = 6.91 \times 10^4/1100 = 62.8$ s (1). $\theta = (\omega_1 + \omega_2)t/2 = 314 \times 62.8/2 = 9.87 \times 10^3$ rad (1). $W = T\theta = 1100 \times 9.87 \times 10^3 = 1.09 \times 10^7$ J (1). $E_k = ½I\omega^2 = ½ \times 220 \times 314^2 = 1.09 \times 10^7$ J, the same, as energy conservation requires (1). The conversion of 3000 rev min^{-1} is the step that carries the whole question. [5]

C3 Her arms are pulled in by internal forces, and no resultant external torque acts on her, so $I\omega$ is conserved (1). Her kinetic energy can be written as $½(I\omega)\omega$; since $I\omega$ is fixed and ω has risen, the kinetic energy must rise too (1). The extra energy is work done by her muscles (1), hauling her arms inward against the outward pull needed to keep them circling (1). Conservation of angular momentum does not imply conservation of energy, and saying so is what separates a full-mark answer from a half one. [4]

C4 $\Delta(I\omega) = 0.40 \times 30 = 12$ kg m^2 s^{-1} (1). $T\Delta t = \Delta(I\omega)$, so $T = 12/2.5 = 4.8$ N m (1). $T = Fr$, so $F = 4.8/0.25 = 19.2$ N (1). All the rotational kinetic energy becomes heat: $E_k = ½ \times 0.40 \times 30^2 = 180$ J (1). Dividing the torque by the wheel's diameter, or quoting the torque itself as the force, are the two standard errors in the third step. [4]