inkMaths

ANSWER BOOK

Angular speed and horizontal circular motion

Further Mechanics 2 · Year FM

6 questions · 25 marks

NAVY EDITION

SECTION A · Warm-up

A1 Velocity is a vector, and although its magnitude is fixed its direction is continually changing, so the velocity is changing and there is an acceleration. B1 for the changing direction of the velocity, B1 for the direction of the acceleration. It points towards the centre of the circle, with magnitude $r\omega^2$ or v^2/r. [2]

A2 $v = r\omega = 1.5 \times 4 =$ **6** m/s. Acceleration $= r\omega^2 = 1.5 \times 16 =$ **24** m/s², directed towards the centre. B1 for the speed, M1 A1 for the acceleration. Using v^2/r gives $36/1.5 = 24$ as well. [3]

SECTION B · Standard

B1 Resolving vertically, the bob has no vertical acceleration, so $T \cos 40° = 0.4(9.8)$ and $T = 3.92/0.766 =$ **5.12** N. Two of the marks are for that equation; writing $T = mg$ with no resolving loses both. [6]

The radius is not the string length. It is $1.2 \sin 40° =$ **0.771** m, and using 1.2 instead is the commonest error in the question.

Resolving horizontally towards the centre, $T \sin 40° = 0.4r\omega^2$. Substituting T and r, the 0.4 and the $\sin 40°$ both cancel, leaving $\omega^2 = g/(l \cos 40°) = 9.8/0.919 = 10.66$, so $\omega = 3.27$ rad/s.

The period is $2\pi/\omega =$ **1.92** s.

M1 A1 for resolving vertically and the tension, B1 for the radius $1.2 \sin 40°$, M1 A1 for the horizontal equation and ω, A1 for the period.

B2 Vertically $R \cos \theta = mg$ and horizontally $R \sin \theta = mv^2/r$. Dividing removes both R and m: $\tan \theta = v^2/(rg)$, so $v^2 = 120 \times 9.8 \times \tan 15° = 315.1$ and $v =$ **17.8** m/s. M1 for both resolved equations, M1 for dividing to reach $\tan \theta = v^2/(rg)$, A1 for $v^2 = 315.1$, A1 for 17.8 m/s. Below that speed friction must act up the slope, and above it down. [4]

B3 On a flat bend the only horizontal force is friction, so it supplies the whole of the radial acceleration: $F = mv^2/r$. At the greatest speed the friction is limiting, $F = \mu R$, and resolving vertically gives $R = mg$, so $F = 0.6mg$. [4]

Setting the two expressions equal, $v^2 = \mu gr = 0.6 \times 9.8 \times 50 = 294$, so $v =$ **17.1** m/s.

M1 for $F = mv^2/r$, B1 for stating that the friction is limiting, M1 for $R = mg$ and $F = \mu R$, A1 for 17.1 m/s.

The mass cancels, so a loaded lorry and an empty car slip at the same speed. State that the friction is at its limiting value; the assumption is worth a mark and is often left unsaid.

SECTION C · Stretch

C1 At the greatest speed the car is on the point of sliding **up** the slope, so the friction acts **down** the slope and is limiting, $F = 0.3R$. Getting that direction the wrong way round gives the least speed instead, and it is the decision the whole question turns on. Draw the forces before writing anything. [6]

Resolving vertically, with no vertical acceleration: $R\cos 20° - 0.3R\sin 20° = mg$.

Resolving horizontally towards the centre: $R\sin 20° + 0.3R\cos 20° = mv^2/60$.

Dividing the second equation by the first removes both R and m, which is the method mark: $v^2/(60g) = (\sin 20° + 0.3\cos 20°)/(\cos 20° - 0.3\sin 20°) = (\tan 20° + 0.3)/(1 - 0.3\tan 20°)$.

With $\tan 20° = 0.364$ that ratio is $0.664/0.891 = 0.745$, so $v^2 = 60(9.8)(0.745) = 438.3$ and $v =$ **20.9** m/s.

B1 for the friction acting down the slope at its limiting value, M1 A1 for resolving vertically, M1 A1 for resolving horizontally, A1 for 20.9 m/s.

As a check, reversing the sign of the friction gives the least speed, 5.82 m/s, and the frictionless design speed of 14.6 m/s lies between the two, as it must.