InkMaths

QUESTION WORKBOOK

Areas, parametric curves and the limit of a sum

Integration · Year 12–13

7 questions · 32 marks

NAVY SCREEN EDITION

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SECTION A · Warm-up

A1 Find the area of the region bounded by the curve $y = 4 - x^2$, the x-axis and the y-axis, in the first quadrant. [3]

A2 Find the area enclosed between the line $y = x$ and the curve $y = x^2$. [4]

SECTION B · Standard

B1 Find the area enclosed between the curves $y = 9 - x^2$ and $y = x^2 + 1$. [5]

B2 The curve C has parametric equations $x = t^3$, $y = 3t^2$, $t \geq 0$. Find the exact area of the region bounded by C, the x-axis and the line $x = 8$, and verify your answer by first converting C to Cartesian form. [6]

B3 Write the limit, as $\delta x \to 0$, of the sum $\Sigma\, x^2\, \delta x$ taken over strips from $x = 1$ to $x = 3$ as an integral, and evaluate it. [3]

SECTION C · Stretch

C1 The curve C has parametric equations $x = 3\cos t$, $y = \sin t$, $0 \leq t \leq \pi/2$. Show that the area of the region bounded by C and the coordinate axes is $3\pi/4$. [5]

C2 The curve C has parametric equations $x = 2 \sin t$, $y = 3 \sin 2t$, $0 \leq t \leq \pi/2$. Show that the area of the region between C and the x-axis is exactly 4. [6]