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ANSWER BOOK

Areas with polar coordinates

Polar coordinates · Year FM

6 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $A = \frac{1}{2}\int r^2\,d\theta$ from α to β. Each thin wedge is nearly a triangle with base $r\,d\theta$ and height r, and a triangle's half shows up in front of the $r^2\,d\theta$. B1 for the formula, B1 for the explanation. [2]

A2 $A = \frac{1}{2}\int 16\,d\theta = 8 \times \pi/3 = 8\pi/3$. M1 for the integral with the right limits, A1 for the area. Check against geometry. The full circle has area 16π, and a sixth of it is $8\pi/3$, matching the sixth of a turn. [2]

SECTION B · Standard

B1 $A = \frac{1}{2}\int 4(1 + \cos\theta)^2\,d\theta$ over 0 to 2π $= 2\int(1 + 2\cos\theta + \cos^2\theta)\,d\theta$. The $\cos\theta$ term dies over a full turn and $\cos^2\theta$ contributes π, so $A = 2(2\pi + \pi) = 6\pi$. M1 for the polar area integral, A1 for the expanded integrand, M1 for the double angle on $\cos^2\theta$, M1 for the limits, A1 for the area. Squaring first and reaching for the double angle is the whole method. [5]

B2 $A = \frac{1}{2}\int 9\cos^2 2\theta\,d\theta = (9/4)\int(1 + \cos 4\theta)\,d\theta$ over the petal. The $\cos 4\theta$ part vanishes across the symmetric limits, leaving $(9/4)(\pi/2) = 9\pi/8$. M1 for the polar area integral, M1 for the double angle, A1 for the reduced integrand, M1 for the limits, A1 for the area. The limits come from $r = 0$. A single loop is bounded by consecutive zeros of $\cos 2\theta$, and taking any wider interval sweeps part of a neighbouring petal as well. [5]

B3 $A = \frac{1}{2}\int 36\sin^2\theta\,d\theta = 9\int(1 - \cos 2\theta)\,d\theta$ over 0 to π $= 9\pi$, which is $\pi \times 3^2$, the circle's area. M1 for the polar area integral, M1 for the double angle, A1 for 9π, B1 for identifying it with the area of a circle of radius 3. The circle is traced once by θ from 0 to π; running to 2π would count it twice. [4]

SECTION C · Stretch

C1 Setting $2(1 + \cos\theta) = 3$ gives $\cos\theta = 1/2$, so the curves meet at $\theta = \pm\pi/3$. For $|\theta| < \pi/3$ the cardioid is outside the circle, so the circle supplies the boundary; beyond $\pi/3$ the cardioid does. Using symmetry in the initial line, the area is $2[\frac{1}{2}\int 9\,d\theta$ from 0 to $\pi/3$ + $\frac{1}{2}\int 4(1 + \cos\theta)^2\,d\theta$ from $\pi/3$ to $\pi]$. The first piece is 3π. For the second, expanding gives $2\int(3/2 + 2\cos\theta + \frac{1}{2}\cos 2\theta)\,d\theta = 2[3\theta/2 + 2\sin\theta + (\sin 2\theta)/4]$, which runs from $\pi/2 + 9\sqrt{3}/8$ at $\pi/3$ to $3\pi/2$ at π, giving $2\pi - 9\sqrt{3}/4$ before doubling. Total $\mathbf{7\pi} - 9\sqrt{3}/2$, about 14.20. M1 A1 for the intersections at $\theta = \pm\pi/3$, B1 for deciding which curve bounds on each side, M1 for the circular sector, M1 A1 for the cardioid piece, A1 for the exact total. Deciding which curve is nearer the pole on each side of the intersection is the whole question; a single integral of either curve is worth nothing. [7]

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