inkMaths

ANSWER BOOK

Arithmetic series

Sequences and series · Year 12–13

7 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $u_{15} = 7 + 14 \times 5 = 77$. M1 for $a + 14d$, A1 for 77. Fourteen steps reach the 15th term, not fifteen. The bracket in $a + (n - 1)d$ is where this topic catches people, and $7 + 15 \times 5 = 82$ is the commonest wrong answer it produces. [2]

A2 $a = 4$ and $d = 7$, so $u_n = 4 + 7(n - 1) = 7n - 3$. Setting $7n - 3 = 200$ gives $n = 29$, a positive whole number, so 200 is the 29th term. B1 for $u_n = 7n - 3$, M1 for setting it equal to 200, A1 for the 29th term. A fractional n would have settled it the other way, and the working that produces it is the proof. Simplifying to $7n - 3$ rather than leaving $4 + 7(n - 1)$ is expected. [3]

SECTION B · Standard

B1 From the 4th term to the 9th is five steps, so $5d = 38 - 18 = 20$ and $d = 4$. Then $a + 3d = 18$ gives $a = 6$, and $u_{25} = 6 + 24 \times 4 = 102$. M1 A1 for $d = 4$, A1 for $a = 6$, A1 for the 25th term. Counting the steps between two given terms beats writing out simultaneous equations, and it removes the commonest error, which is dividing the difference by six rather than five. [4]

B2 $a = 8$ and $d = 3$, so $S_{30} = 15 \times (16 + 29 \times 3) = 15 \times 103 = 1545$. The last-term form agrees, since $l = 8 + 29 \times 3 = 95$ and $15 \times (8 + 95) = 1545$. Two dressings of one formula give a check that costs a single line. M1 for the sum formula with a, d and n in place, A1 for the correct substitution, A1 for 1545. Marks go to the substitution as much as the total, so write the formula out with a, d and n in place before reaching for the calculator. [3]

B3 Write $S = 1 + 2 + \ldots + n$ forwards and backwards and add the two lines: each column pairs to $n + 1$, and there are n columns, so $2S = n(n + 1)$ and $S = n(n + 1)/2$. For $n = 200$: $200 \times 201/2 = 20\,100$. M1 for reversing and adding, A1 for $2S = n(n + 1)$, A1 for the printed result, B1 for the value of the sum. The pairing argument is the required proof, and it is two lines long. [4]

B4 The deposits are arithmetic with $a = 20$ and $d = 4$, so $S_{24} = 12 \times (40 + 23 \times 4) = 12 \times 132 = £1584$. M1 for the sum formula with $a = 20$ and $d = 4$, A1 for the substitution, A1 for the total. This is a sum rather than a term, because every month's deposit stays in the pot. Answering £112, the 24th deposit alone, is the misreading the phrase total saved is there to prevent. [3]

SECTION C · Stretch

C1 $S_n = n(2 \times 5 + 4(n - 1))/2 = n(2n + 3)$ must exceed 1000. Trying the boundary: $S_{21} = 945$, too small, and $S_{22} = 1034$, past the mark. So 22 terms are needed. M1 for S_n in terms of n, M1 for comparing it with 1000, A1 for locating the boundary, A1 for 22 terms. Solving the quadratic $2n^2 + 3n - 1000 > 0$ gives $n > 21.6$, the same verdict; either route must end with whole terms, rounded up. [4]