inkMaths

ANSWER BOOK

Calculus with inverse trigonometric functions

Further calculus · Year FM

6 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $\frac{d}{dx}(\arcsin x) = \mathbf{1/\sqrt{(1 - x^2)}}$ and $\frac{d}{dx}(\arctan x) = \mathbf{1/(1 + x^2)}$. B1 B1. Neither derivative contains a trigonometric function, which is what makes the two results so useful read backwards, as standard integrals. [2]

A2 Differentiate implicitly to get $\sec^2 y\,(dy/dx) = 1$. Since $\sec^2 y = 1 + \tan^2 y = 1 + x^2$, $dy/dx = \mathbf{1/(1 + x^2)}$. M1 for the implicit differentiation, M1 for replacing $\sec^2 y$ by $1 + x^2$, A1 for the printed result. The identity is the whole point of the derivation; it turns the trigonometry in the denominator into algebra in x, and without it the answer is left as $1/\sec^2 y$ and scores no accuracy mark. [3]

SECTION B · Standard

B1 The chain rule brings the inner derivative to the front, so $dy/dx = 2/\sqrt{(1 - 4x^2)}$. Dropping that factor of 2 is the single most common slip here, and it loses the method mark. [3]
At $x = 1/4$: $2/\sqrt{(1 - 1/4)} = 2/\sqrt{(3/4)} = \mathbf{4/\sqrt{3}}$, about 2.31. M1 A1 for the derivative, A1 for the gradient.

B2 Match the form $a^2 + x^2$ with $a = 2$, so the integral is $(1/2)\arctan(x/2)$. The $1/a$ in front belongs to the arctan pattern; omitting it doubles the final answer. [4]
At the limits: $(1/2)(\arctan 1 - \arctan 0) = (1/2)(\pi/4) = \mathbf{\pi/8}$. M1 for the arctan form, A1 for the antiderivative, M1 for substituting the limits, A1 for the exact value.

B3 Match the form $a^2 - x^2$ under the root with $a = 3$, so the integral is $\arcsin(x/3)$, with no factor in front. The arcsin pattern and the arctan pattern differ in exactly that respect. [4]
At the limits: $\arcsin(1/2) - \arcsin 0 = \mathbf{\pi/6}$. M1 for the arcsin form, A1 for the antiderivative, M1 for substituting the limits, A1 for the exact value.

SECTION C · Stretch

C1 Neither standard form fits as it stands, so split the numerator into the derivative of the denominator plus a constant: $2x + 4 = (2x + 2) + 2$. That split is the first method mark, and an attempt at a single arctan or a single logarithm gets nowhere. [6]
The first piece integrates by recognition, since its numerator is exactly the derivative of the denominator, giving $\ln(x^2 + 2x + 5)$.
For the second piece, complete the square: $x^2 + 2x + 5 = (x + 1)^2 + 4$, so $\int 2/((x + 1)^2 + 4)\,dx = 2 \times (1/2) \arctan((x + 1)/2) = \arctan((x + 1)/2)$.
Evaluating from -1 to 1: the logarithm gives $\ln 8 - \ln 4 =$ **ln** 2, and the arctan gives $\arctan 1 - \arctan 0 =$ **π/4**. The total is **ln** $2 + \pi/4$, about 1.479, as required.
M1 A1 for the split of the numerator, A1 for the logarithm term, M1 for completing the square, A1 for the arctan term, A1 for the printed result. Any quadratic denominator with no real roots yields this same pair of pieces, a logarithm and an arctan.