inkMaths

ANSWER BOOK

Centre of mass of a discrete distribution

Further Mechanics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

A1 Taking moments about the origin, each particle contributes its mass times its distance, and the total must equal the whole mass placed at the centre of mass. The masses are therefore the weights, so the point is pulled towards the heavier particles. B1 for the moment argument, B1 for the pull towards the heavier particles. Only when all the masses are equal does it reduce to the plain average. [2]

A2 $\Sigma mx = 3(0) + 5(4) + 2(10) = 0 + 20 + 20 = 40$, and the total mass is 10 kg. So $x(G) = 40/10 = \mathbf{4}$. M1 for the sum of the moments, A1 for 40 over a total mass of 10, A1 for the position. It lies between the outer masses as it must. [3]

SECTION B · Standard

B1 Total mass 10 kg. $\Sigma mx = 0 + 4 + 6 + 0 = 10$, so $x(G) = \mathbf{1}$. $\Sigma my = 0 + 0 + 9 + 12 = 21$, so $y(G) = \mathbf{2.1}$. M1 A1 for the x coordinate, M1 A1 for the y coordinate. The point (1, 2.1) sits towards the top of the rectangle, pulled there by the two heavier masses at $y = 3$. [4]

B2 The formula gives $(4 \times 2 + 8m)/(4 + m) = 5$, so $8 + 8m = 20 + 5m$ and $3m = 12$, giving $m = \mathbf{4}$ kg. M1 for the moment equation, A1 for the linear equation in m, M1 for solving, A1 for the mass. Checking: $(8 + 32)/8 = 5$. The centre is halfway between them, which is what equal masses always produce. [4]

B3 Supported at that point it balances, because the moments of the weights about it cancel. And suspended freely from any point, it hangs with the centre of mass vertically below that point, since otherwise the weight would have a moment about the pivot and the body would turn. B1 for balancing at that point, B1 B1 for hanging with the centre of mass below the point of suspension, with a reason. [3]

SECTION C · Stretch

C1 Take moments about each axis in turn, keeping the negative coordinates negative. Dropping a sign here is the error the question is built around. [6]

For the three original particles: $\Sigma mx = 2(1) + 3(-2) + 5(3) = 2 - 6 + 15 = 11$, and $\Sigma my = 2(4) + 3(0) + 5(-2) = 8 + 0 - 10 = -2$, with total mass 10 kg.

With the fourth particle included, the total mass is $10 + m$ and
$x(G) = (11 + 5m)/(10 + m)$, $y(G) = (-2 + 6m)/(10 + m)$.

The condition $y(G) = x(G)$ has the same denominator on both sides, so it cancels. That cancellation is the method mark; multiplying out and solving a quadratic wastes time and invites errors.

So $11 + 5m = -2 + 6m$, giving **m** = 13 kg.

M1 A1 for the moments of the first three particles, M1 for the two coordinates with the fourth included, M1 for setting them equal and cancelling the denominator, A1 for the linear equation, A1 for the mass.

Check: the total mass is 23 kg, and both coordinates come to $76/23 \approx 3.30$, so the centre of mass is at (3.30, 3.30), which does lie on $y = x$.