inkMaths

ANSWER BOOK

Centres of mass of plane figures and frameworks

Further Mechanics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

A1 A rectangle's centre of mass is at its centre, where the diagonals cross. A triangle's is at the centroid, a third of the way from each side towards the opposite vertex. Any axis of symmetry must contain the centre of mass, so a symmetric shape needs only one coordinate calculating. B1 for the two standard positions, B1 for the symmetry argument. [2]

A2 Split the shape into the two rectangles and treat each as a particle at its own centre. Areas 12 and 8, total 20, with centres (3, 1) and (1, 4). [3]
$x(G) = (12 \times 3 + 8 \times 1)/20 = 44/20 =$ **2.2**.
$y(G) = (12 \times 1 + 8 \times 4)/20 = 44/20 =$ **2.2**.
M1 for the areas acting at their own centres, A1 for the x coordinate, A1 for the y coordinate.
The L is symmetric about the line $y = x$, so the two coordinates were bound to agree. That makes a quick check on the arithmetic.

SECTION B · Standard

B1 Areas 16π and 4π, leaving 12π. By symmetry the centre of mass lies on the x-axis. Treating the hole as a negative area: $x(G) = (16\pi \times 0 - 4\pi \times 2)/(12\pi) = -8/12 =$ **$-2/3$**. B1 for the symmetry, M1 for treating the hole as a negative area, A1 for the moment equation, A1 for the position. It has moved away from the hole, as it must, and the π has cancelled throughout. [4]

B2 Lengths 6, 4 and 4, total 14, acting at (3, 0), (6, 2) and (0, 2). $x(G) = (18 + 24 + 0)/14 = 42/14 =$ **3**, which symmetry about $x = 3$ confirms. $y(G) = (0 + 8 + 8)/14 = 16/14 =$ **$8/7$** ≈ 1.14. M1 for the lengths acting at their midpoints, A1 for the x coordinate, M1 A1 for the y coordinate. It sits below the middle because the long rod lies along the bottom. [4]

B3 The lamina has its mass spread over the area, so each element is weighted by area and the answer is the centroid of the region. The framework has its mass along the perimeter, so each rod is weighted by its length and acts at its own midpoint. Different distributions of the same total mass give different balance points. B1 for the lamina weighted by area, B1 for the framework weighted by rod length, B1 for each rod acting at its own midpoint. [3]

SECTION C · Stretch

C1 For a framework the mass of each rod is proportional to its **length**, and each rod acts at its own **midpoint**. Setting the work out as a table of length against midpoint is what earns the method marks. [6]

The three rods have lengths 4, 3 and 5, the last by Pythagoras, giving a total of 12. Their midpoints are (2, 0), (0, 1.5) and (2, 1.5).

$x(G) = [4(2) + 3(0) + 5(2)]/12 = 18/12 = \mathbf{1.5}$.

$y(G) = [4(0) + 3(1.5) + 5(1.5)]/12 = 12/12 = \mathbf{1}$.

For the lamina the mass is spread over the area, and the centre of mass is the centroid, the average of the three vertices: $x(G) = (0 + 4 + 0)/3 = \mathbf{4/3}$ and $y(G) = (0 + 0 + 3)/3 = \mathbf{1}$.

The two heights agree at $y = 1$, which is a coincidence of these particular numbers rather than a rule. The x coordinates differ, 1.5 against 4/3, because the framework's heaviest rod is the hypotenuse, whose midpoint is at $x = 2$, and that pulls the framework to the right.

M1 for the lengths and their midpoints, A1 for the x coordinate, A1 for the y coordinate, M1 A1 for the centroid of the lamina, B1 for the comparison.

Using the centroid formula on a framework is the mistake this question exists to catch. It is only valid for a lamina.

