inkMaths

ANSWER BOOK

Comparing two normal means

Further Statistics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

A1 Each mean has variance σ^2/n, so $50/25 = 2$ and $30/25 = 1.2$. Adding gives 3.2, so the standard error is $\sqrt{3.2}$ = **1.789** to three decimal places. M1 for adding the two variances of the means, A1 for the standard error. The variances are added, never subtracted, even though the means are subtracted. [2]

A2 Standard error = $\sqrt{(9/36 + 16/49)} = \sqrt{(0.25 + 0.3265)} = \sqrt{0.5765}$ = **0.759**. The difference of means is 1.8, so $z = 1.8/0.759$ = **2.37**. M1 A1 for the standard error, A1 for the statistic. [3]

SECTION B · Standard

B1 H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$, where μ_1 and μ_2 are the **population** means. Two-tailed at 5%, so the critical values are ±1.96. The statistic 2.37 exceeds 1.96 and lies in the critical region, so **reject** H_0. There is evidence at the 5% level that the two population means differ, with the first appearing larger. B1 for the hypotheses in terms of population means, B1 for the critical values, M1 for the comparison, A1 for the conclusion in context. Hypotheses written about $\bar{x}_1$ and $\bar{x}_2$ score nothing, since the sample means are known and nothing is being claimed about them. [4]

B2 The point estimate is 1.8 and the standard error 0.759, so the interval is 1.8 ± 1.96 × 0.759 = 1.8 ± 1.488, that is **(0.31,** 3.29). M1 for the form estimate plus or minus z times the standard error, A1 for the half-width, A1 for the interval, B1 for the comparison with the test. Zero lies outside, which is exactly the verdict the two-tailed test at 5% reached. The interval says more: the difference could plausibly be anywhere from a third of a unit to over three. [4]

B3 Replace each σ^2 by the sample variance s^2 from that sample. The Central Limit Theorem keeps the sample means approximately normal whatever the population shape, and with large samples the estimated variances are close enough to the real ones for the normal critical values to remain usable. The conclusion becomes approximate rather than exact, and this should be stated. B1 for replacing each variance by its sample estimate, B1 for the Central Limit Theorem, B1 for saying the result is approximate. [3]

SECTION C · Stretch

C1 $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 > \mu_2$. The statistic is 2.371, which exceeds 2.3263, so **reject** H_0 at the 1% level as well. [6]
The test rejects when the difference d satisfies $d/0.7593 > 2.3263$, that is $d > 2.3263 \times 0.7593 =$ **1.766**.
The observed 1.8 clears that by only 0.034. A difference of 1.76 would have been significant at 5% but not at 1%, so the two levels would have disagreed. B1 for the one-tailed hypotheses, M1 for the comparison with 2.3263, A1 for the conclusion in context, M1 A1 for the smallest significant difference, B1 for the comment on how marginal it is. Report a statistic this close to its critical value as marginal rather than as a firm verdict, and quote the level with every conclusion.