inkMaths

ANSWER BOOK

Conditional probability

Statistics · Year 12–13

6 questions · 20 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $P(A|B) = P(A \cap B)/P(B) = 0.18/0.6 = 0.3$, the overlap measured as a share of B's world rather than of the whole. Independence means conditioning on B changes nothing, so $P(A)$ would have to be 0.3 as well. M1 for the conditional formula, A1 for 0.3, B1 for the value $P(A)$ would need under independence. Dividing $P(A \cap B) = P(A)P(B)$ through by $P(B)$ turns the product form of independence into this conditional form, so the two definitions say one thing. [3]

A2 Multiply along the branch. $P(A \cap B) = P(B) \times P(A|B) = 0.4 \times 0.25 = 0.1$. M1 for the product along the branch, A1 for 0.1. This is the conditional formula rearranged, and it is what every tree diagram does. The second-stage probabilities on a tree are already conditional on the first stage, so multiplying along a path gives the probability of the whole path. [2]

SECTION B · Standard

B1 Two routes end in a blue second draw. Red then blue is $4/7 \times 3/6 = 12/42$, and blue then blue is $3/7 \times 2/6 = 6/42$. Adding them gives $18/42 = 3/7$. Both red is a single route, $4/7 \times 3/6 = 12/42 = 2/7$. The second branch carries a denominator of 6 because a counter has already gone, and that shift is what 'without replacement' means. M1 for the products along the branches, M1 for adding the two routes, A1 for the first probability, A1 for the second. [4]

B2 P(first red | second blue) = P(red then blue)/P(second blue) = $(12/42)/(18/42) = 12/18 = 2/3$. Conditioning on the later event to ask about the earlier one feels backwards, but the formula has no sense of time. It compares an overlap with a whole and nothing more. M1 for the conditional formula, A1 for the correct pair of probabilities, A1 for $2/3$. The denominator has to be P(second blue). Putting P(first red) underneath instead is the standard slip and answers a different question. [3]

B3 Reading along the adult row, P(attends | adult) = $45/60 = 3/4$. The attenders number $45 + 10 = 55$, so P(adult | attends) = $45/55 = 9/11$. B1 for the first probability, M1 for the 55 attenders, A1 for the second. Same overlap of 45 members, two different denominators. The event written after the bar decides which world the fraction lives in, and swapping the two earns no marks even though the numerator is identical. [3]

SECTION C · Stretch

C1 Draw the tree with the machine first, then condition. P(A and faulty) = 0.6 × 0.03 = 0.018 and P(B and faulty) = 0.4 × 0.08 = 0.032, so P(faulty) = 0.018 + 0.032 = 0.05. Then P(B | faulty) = 0.032/0.05 = 0.64. M1 A1 for the two products, M1 for adding them to reach the total probability of a fault, M1 for dividing the right product by that total, A1 for 0.64. Look at what the answer says. Machine B makes only 40% of the components yet accounts for 64% of the faulty ones, because the evidence has reweighted the branches. A patient who reads a 99% accurate positive test as a 99% chance of illness has run the same calculation backwards, quoting P(positive | ill) when the question asks for P(ill | positive). The direction of the bar fixes the denominator, and the denominator fixes the answer. [5]