inkMaths

ANSWER BOOK

# Conic sections

Further Pure 1 · Year FM

7 questions · 27 marks

NAVY EDITION


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## SECTION A · Warm-up

**A1** Parabola: $(at^2, 2at)$. Rectangular hyperbola: $(ct, c/t)$. B1 B1 for the two parametrisations. Both turn a two-variable problem into algebra in a single parameter, and almost every tangent and normal question on this topic starts by using them. [2]

**A2** $4a = 20$, so $a = 5$. The focus is $(5, 0)$ and the directrix is $x = -5$. B1 for $a = 5$, B1 for the focus, B1 for the directrix. Quartering rather than halving the coefficient is the step most often slipped. [3]

## SECTION B · Standard

**B1** $a = 4$ and $b^2 = 7$. From $b^2 = a^2(1 - e^2)$: $7 = 16(1 - e^2)$, so $e^2 = 9/16$ and $e = 3/4$. Foci $(\pm ae, 0) = (\pm 3, 0)$; directrices $x = \pm a/e = \pm 16/3$. M1 for using $b^2 = a^2(1 - e^2)$, A1 for $e = 3/4$, A1 for the foci, A1 for the directrices. The minus sign inside the bracket is what distinguishes the ellipse formula from the hyperbola's. [4]

**B2** Here $b^2 = a^2(e^2 - 1)$: $9 = 16(e^2 - 1)$, so $e^2 = 25/16$ and $e = 5/4$. Foci $(\pm 5, 0)$. From $(4, 0)$ the distances are 1 and 9, differing by $8 = 2a$. M1 for using $b^2 = a^2(e^2 - 1)$, A1 for $e = 5/4$, A1 for the foci, B1 for the difference $2a$. A hyperbola holds the difference of focal distances constant, as an ellipse holds the sum. [4]

**B3** Every point of the conic satisfies distance to the focus = $e$ × distance to the directrix. When $e < 1$ the curve closes into an ellipse; $e = 1$ balances it into a parabola; $e > 1$ opens it into a hyperbola's two branches. B1 for the focus-directrix property, B1 B1 for the three cases of $e$. One definition with a dial produces all three. [3]

**B4** $16^2 = 256$ and $16 \times 16 = 256$, so P is on the curve. Here $4a = 16$ and $a = 4$, so the focus is $(4, 0)$ and the directrix is $x = -4$. The distance from P to the focus is $\sqrt{(12^2 + 16^2)} = 20$, and the distance to the directrix is $16 + 4 = 20$. Equal, exactly as $e = 1$ demands. B1 for the verification, B1 for the focus and directrix, M1 for a distance, A1 for both distances equal to 20. [4]

## SECTION C · Stretch

**C1** Here a = 4, so S is (4, 0) and d is $x = -4$. In parametric form $P = (at^2, 2at)$ gives $2at = 16$ with $a = 4$, so $t = 2$ and $at^2 = 16$ as required. Differentiating $y^2 = 16x$ implicitly, $2y(dy/dx) = 16$, so at P the gradient is $16/32 = 1/2$ and the tangent is $y - 16 = \frac{1}{2}(x - 16)$, that is $2y = x + 16$. Setting $x = -4$ gives $y = 6$, so **Q(−4,** 6). Then SP = (12, 16) and SQ = (−8, 6), and their scalar product is $-96 + 96 = 0$, so **angle** PSQ = 90°. B1 for S and d, M1 for differentiating to find the gradient at P, A1 for the tangent $2y = x + 16$, M1 for putting $x = -4$, A1 for Q(−4, 6), M1 for the scalar product of SP and SQ, A1 for the conclusion. This holds for every point of every parabola, not only for P; repeating the working with $(at^2, 2at)$ gives $SP \cdot SQ = 0$ in general. [7]