inkMaths

ANSWER BOOK

Determinants and inverses

Matrices · Year FM

7 questions · 24 marks

NAVY EDITION

SECTION A · Warm-up

A1 det $= 3 \times 2 - 1 \times 1 = 5$. Areas are scaled by 5, and the positive sign means orientation is preserved, so no reflection is hiding in the transformation. B1 for the determinant, B1 for the area scale factor with orientation preserved. [2]

A2 Swap the diagonal, negate the off-diagonal, divide by the determinant 5: $(1/5) \times$ rows $(2, -1)$, $(-1, 3)$. Multiplying back gives $(1/5) \times$ rows $(5, 0)$, $(0, 5) = I$, as required. M1 for swapping and negating, A1 for dividing by 5, B1 for the multiplication giving I. [3]

SECTION B · Standard

B1 Expand along the top row: $1 \times (1 \times 1 - 4 \times 0) - 2 \times (3 \times 1 - 4 \times 2) + 0 = 1 - 2 \times (-5) = 11$. M1 for expanding along a row, A1 for the two by two minors, A1 for 11. The zero in the top row kills a third of the work; expand along whichever row or column carries the most zeros. [3]

B2 det $= 3 \times 2 - 0 \times 1 = 6$, so areas scale by 6 and the image has area 24. B1 for the determinant, B1 for 24. The determinant is the area scale factor whatever the shape, so no information about the original outline is needed. [2]

B3 Singular means det $= 0$, so $k - 6 = 0$ and $k = 6$. The transformation then squashes the plane onto a line, area scale factor zero, and no inverse can exist because the squash cannot be undone. M1 for setting the determinant to zero, A1 for $k = 6$, B1 for the collapse onto a line. [3]

SECTION C · Stretch

C1 Multiply: $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AA^{-1} = I$, and likewise on the other side, so $B^{-1}A^{-1}$ is the inverse of AB. The order reverses because undoing a sequence means undoing the last action first, like unwrapping layers in the reverse order they went on. M1 for forming the product, A1 for reducing it to I, A1 for the other side as well, B1 for the reason for the reversal. [4]

C2 det N = 11 from the earlier expansion, so the inverse exists. The matrix of minors has rows (1, -5, -2), (2, 1, -4) and (8, 4, -5). Applying the alternating sign pattern gives the cofactors, rows (1, 5, -2), (-2, 1, 4) and (8, -4, -5). Transposing gives the adjugate, rows (1, -2, 8), (5, 1, -4) and (-2, 4, -5), and dividing by 11 gives $\mathbf{N}^{-1}$ = (1/11) × rows (1, -2, 8), (5, 1, -4), (-2, 4, -5). B1 for det N = 11, M1 for the matrix of minors, A1 for the minors, M1 for the alternating signs, M1 for transposing, A1 for the adjugate, A1 for the inverse. Multiplying N by it returns 11I before the division, which is the check worth doing. The transpose is the step candidates omit, and it only shows up in the answer when the matrix is not symmetric. [7]