InkMaths

QUESTION WORKBOOK

Diagonalisation and the Cayley-Hamilton theorem

Further Pure 2 · Year FM

6 questions · 23 marks

PRINT EDITION

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SECTION A · Warm-up

A1 The matrix M has rows (4, 1) and (2, 3), with eigenvalues 5 and 2 and corresponding eigenvectors (1, 1) and (1, -2). Write down matrices P and D such that $M = PDP^{-1}$. [2]

A2 State the Cayley-Hamilton theorem, and verify it for the matrix M with rows (4, 1) and (2, 3). [3]

SECTION B · Standard

B1 The matrix M with rows (4, 1) and (2, 3) has eigenvalues 5 and 2 with eigenvectors (1, 1) and (1, -2). Use a diagonalisation to find M^5. [5]

B2 The matrix M has rows (4, 1) and (2, 3). Use the Cayley-Hamilton theorem to express M^{-1} in terms of M and I, and hence evaluate M^{-1}. [4]

B3 Give a 2 by 2 matrix that cannot be diagonalised, and explain what goes wrong. [3]

SECTION C · Stretch

C1 The matrix M with rows (4, 1) and (2, 3) has eigenvalues 5 and 2. Show that for every positive integer n, $M^n = a_nM + b_nI$ where $a_n = (5^n - 2^n)/3$, and find b_n. Hence evaluate M^6. [6]