



ANSWER BOOK

Differentiating powers of x

Differentiation · Year 12–13

7 questions · 20 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $dy/dx = 5x^4 - 6x$. M1 A1. The constant 7 contributes nothing. A flat line has gradient zero, wherever it sits. [2]
- A2** Rewrite as $x^{1/2}$. The power rule then gives $(1/2)x^{-1/2}$, which is $1/(2\sqrt{x})$. M1 for the rewrite and the power rule, A1 for $1/(2\sqrt{x})$. The rewrite comes first, because the rule cannot see a root sign. Leaving the answer as $\frac{1}{2}x^{-1/2}$ ignores the instruction and drops the final accuracy mark. [2]

SECTION B · Standard

- B1** Rewrite the fraction as $-2x^{-1}$. Then $dy/dx = 12x^2 + 2x^{-2} = 12x^2 + 2/x^2$. M1 for rewriting as a power, A1 for $12x^2$, A1 for the second term. Bringing down the -1 multiplies the -2 to give +2, and that positive sign is what the question exists to test. An answer ending in $-2/x^2$ loses the accuracy mark. [3]
- B2** Rewrite as $6x^{1/2} + 3x^{-2} - 5x$, then apply the rule term by term. That gives $3x^{-1/2} - 6x^{-3} - 5$, or $3/\sqrt{x} - 6/x^3 - 5$ in the original notation. M1 for rewriting all three terms as powers, A1 A1 for the derivative. Every term has to become a power of x before the rule touches it. The commonest error is reading $3/x^2$ as if the index were positive, which flips the sign of the middle term. [3]
- B3** Expand first, so $y = x^5 - 4x^2$ and $dy/dx = 5x^4 - 8x$. M1 for expanding, A1 for the derivative. The power rule handles sums, not products. Differentiating each bracket and multiplying gives $2x \times 3x^2 = 6x^3$, which is wrong. [2]
- B4** Divide through first. $y = 2x - 5/x = 2x - 5x^{-1}$, so $dy/dx = 2 + 5x^{-2} = 2 + 5/x^2$. M1 for dividing through, A1 for the derivative. A quotient with a single-term denominator splits by division, and no quotient rule is needed here. [2]

SECTION C · Stretch

- C1** Expand the square before dividing. $(2\sqrt{x} - 3)^2 = 4x - 12\sqrt{x} + 9$, so dividing every term by x gives $y = 4 - 12x^{-1/2} + 9x^{-1}$. Differentiating gives $dy/dx = 6x^{-3/2} - 9x^{-2}$, as required. Setting that equal to zero, $6x^{-3/2} = 9x^{-2}$. Multiply both sides by x^2 to get $6x^{1/2} = 9$, so $\sqrt{x} = 3/2$ and $x = 9/4$. M1 for expanding the square, M1 for dividing every term by x, A1 for the printed derivative, M1 for setting the derivative to zero, dM1 for clearing the negative indices, A1 for $x = 9/4$. Three separate traps sit in this one question. Squaring the bracket as $4x + 9$ loses the middle term, dividing only the first term by x wrecks the indices, and $-1/2 - 1$ has to come out as $-3/2$ rather than $-1/2$. [6]