inkMaths

ANSWER BOOK

Equilibrium, suspension, toppling and sliding

Further Mechanics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

A1 Vertically below the point of suspension. If it were not, the weight would have a moment about the pivot and the body would turn until it was. B1 for the position, B1 for the moment argument. That single fact fixes the angle at which any suspended lamina hangs. [2]

A2 The line from the pivot to G makes an angle arctan(2/3) with the vertical, so the edge tilts by **33.7°**. B1 for taking the line to G as vertical, M1 for $\tan\theta = 2/3$, A1 for the angle. Dividing the other way round would give 56.3°, the angle to the horizontal, so state which angle you have found. [3]

SECTION B · Standard

B1 Toppling: the base half-width is 0.25 and the height of G is 0.6, so $\tan\theta = 0.25/0.6 = 0.4167$ and $\theta =$ **22.6°**. Sliding: $\tan\theta = \mu = 0.5$, so $\theta =$ **26.6°**. The smaller angle comes first, so the block **topples** at 22.6°. M1 for $\tan\theta = 0.25/0.6$, A1 for 22.6°, B1 for the sliding angle, A1 for choosing the smaller of the two. [4]

B2 Toppling begins when $\tan\theta = a/h$, where $a = 0.25$ is the half-width of the base and $h = 0.6$ is the height of the centre of mass. Sliding begins when $\tan\theta = \mu$.
The two coincide when $\mu = a/h = 0.25/0.6 =$ **0.4167**. Setting the two tangents equal, rather than the two angles, is the quickest route and is worth the method mark.
Below that value the friction limit is reached at the smaller angle, so the block **slides**. Above it the line of the weight leaves the base first, so the block **topples**. M1 for equating the two tangents, A1 for 0.4167, B1 B1 for the two cases.
A smoother surface therefore protects a tall block from falling over, which runs against most people's intuition. [4]

B3 The normal reaction acts somewhere within the area of contact. For equilibrium its moment about the centre of mass must balance, which is only possible if the vertical through the centre of mass meets the base. Once it falls outside, no position of the reaction can balance the moments and the block turns. Height matters only through where that vertical line lands. B1 for the reaction lying within the base, B1 for moments balancing only while the vertical through G meets it, B1 for the turn once it does not. [3]

SECTION C · Stretch

C1 Test the two possibilities separately and compare the forces they need. [6]
Working out only one of them is what loses this question.
Sliding. Vertically $R = 200$. The block slides when P reaches the limiting friction $\mu R = 0.4(200) =$ **80** N.
Toppling. The block turns about the bottom edge furthest from P. At that instant the normal reaction has moved to that edge, so it has no moment about it, and only P and the weight remain. Taking moments about the edge: P acts at the top, a distance 1.0 m up, and the weight acts through the centre, 0.3 m horizontally from the edge.
So $P(1.0) = 200(0.3)$, giving $P =$ **60** N.
Sixty newtons is reached before eighty, so the block **topples** first, at $P = 60$ N.
B1 for $R = 200$, M1 A1 for the sliding value of 80 N, M1 for moments about the edge, A1 for $P = 60$ N, A1 for the comparison.
Saying where the normal reaction acts is worth a mark on its own, and taking moments about the centre of mass instead of the edge is the usual wrong start.