inkMaths

ANSWER BOOK

Exponential functions and e

Exponentials and logarithms · Year 12

7 questions · 21 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The intercept is (0, 1), since $3^0 = 1$. At $x = 2$, $y = 9$. The asymptote is $y = 0$, the x-axis, which the curve approaches on the left without ever touching. B1 B1 B1 for the three answers. Every curve $y = a^x$ passes through (0, 1) whatever a is, so that intercept is worth remembering rather than working out. [3]

A2 Every exponential is positive everywhere: 5^x takes every value above zero and no others. The curve hugs the x-axis as x heads left but never reaches it, so zero is approached, not achieved. B1 for the range being strictly positive, B1 for the asymptote argument. [2]

SECTION B · Standard

B1 Differentiating e^{kx} multiplies by k, so the gradient function is $4e^{4x}$. At $x = 0$ the gradient is $4e^0 = 4$, and at $x = 0.5$ it is $4e^2 = 29.6$ to 3 significant figures, four times the height there. B1 for the gradient function, B1 for 4, B1 for 29.6. The factor of 4 is the only mark at risk; writing e^{4x} unchanged is the error the chain rule exists to prevent. [3]

B2 The gradient function is $-2e^{-2x}$, negative everywhere: the curve falls at every point. At $x = 1$, $y = e^{-2} = 0.135$ to 3 significant figures. As x grows the curve decays towards the asymptote $y = 0$, steeply at first and ever more gently. B1 for the gradient function, B1 for 0.135, B1 for the decay to the asymptote. [3]

B3 $P = 500 \times 3^t$, since each unit step in t multiplies the population by the same ratio 3. After 4 hours, $P = 500 \times 81 = 40\,500$. B1 for the model, M1 for substituting $t = 4$, A1 for the value after four hours. The starting value belongs in front and the growth factor in the base, so $P = 500t^3$ or $P = 3 \times 500^t$ both fail the first check, which is that $P = 500$ at $t = 0$. [3]

SECTION C · Stretch

C1 The gradient function is $5 \times 0.2e^{0.2x} = 0.2 \times 5e^{0.2x} = 0.2y$: the rate of change is proportional to the amount, with constant 0.2. At $x = 5$ the height is $5e = 13.6$ and the gradient is $0.2 \times 5e = e = 2.72$. M1 for differentiating, A1 for the link to 0.2y, A1 for 13.6, A1 for 2.72. This proportionality is the property that makes e the modelling base. [4]

C2 A gradient of exactly 1 at (0, 1) must belong to some base between 2 and 3, and that base is $e = 2.718\ldots$. The defining property is that $y = e^x$ is its own gradient function: at every point the gradient equals the height, and (0, 1) is only the most convenient place to notice it. B1 for the base lying between 2 and 3, B1 for $e = 2.718$, B1 for the self-derivative property. [3]