



QUESTION WORKBOOK

Flows in networks: cuts and capacity

Decision Mathematics 2 · Year FM

6 questions · 22 marks

PRINT EDITION

SECTION A · Warm-up

- A1** A cut crosses arcs of capacity 9 and 6 towards the sink and an arc of capacity 5 back towards the source. Find its capacity and explain the treatment of the third arc. [2]

- A2** Explain why the capacity of any cut is an upper bound for the flow through the network. [2]

SECTION B · Standard

- B1** A network has SA 12, SB 9, AC 7, AD 6, BD 8, CT 10, DT 11, all arcs directed away from S and towards T. Find the capacity of the cut round the source, the cut round the sink, and the cut separating $\{S, A, B, D\}$ from $\{C, T\}$. [5]

- B2** A network has arcs SA 12, SB 9, AC 7, AD 6, BD 8, CT 10 and DT 11, all directed away from S and towards T. Find the capacity of the cut separating $\{S, B\}$ from $\{A, C, D, T\}$, and explain why a cut of capacity 20 tells you less than a cut of capacity 18.

[3]

- B3** A different network has two factories and three shops. Describe how to bring it into the standard one-source, one-sink form.

[3]

SECTION C · Stretch

- C1** A network has arcs SA 12, SB 9, AC 7, AD 6, BD 8, CT 10 and DT 11, all directed away from S and towards T, and vertex D can handle at most 8 units in total. Explain how to model the restriction on D, find the capacity of the cut separating $\{S, A, B, D\}$ from $\{C, T\}$, and by stating a flow of that value prove that it is the maximum.

[7]