inkMaths

ANSWER BOOK

Functions in modelling

Algebra and functions · Year 12–13

7 questions · 24 marks

NAVY EDITION

SECTION A · Warm-up

A1 Equal steps added: linear. Equal factors: exponential. Repeating on a fixed cycle: trigonometric. B1 B1 B1 for the three families. Diagnosis comes before fitting; the behavioural signature picks the family, and only then do constants get estimated. [3]

A2 The sine runs between ±1, so h runs from 6 - 1.8 = 4.2 m up to 6 + 1.8 = 7.8 m. B1 B1 for the two depths. Centre line 6, swing 1.8: the two numbers in the model are exactly these two facts. [2]

SECTION B · Standard

B1 The sine completes a cycle when 30t reaches 360, so the period is 12 hours. High tide needs sin $(30t)^\circ = 1$, first at 30t = 90, so t = 3 hours. B1 for the period, M1 for 30t = 90, A1 for t = 3. Both answers come from reading the coefficient, not from any solving. [3]

B2 At t = 0 the value is 400 + 5600 = £6000. As t grows the exponential dies away, leaving the £400 floor: scrap value. At t = 5: $V = 400 + 5600e^{-1}$ = £2460. B1 for £6000, B1 for £400, M1 for substituting t = 5, A1 for £2460. The two constants carry the two ends of the story: start, and long run. [4]

B3 $400 + 5600e^{-0.2t} = 1200$ gives $e^{-0.2t} = 800/5600 = 1/7$. Taking logs: 0.2t = ln 7, so t = 9.73 years. M1 for isolating the exponential, A1 for 1/7, M1 for taking logs, A1 for 9.73 years. The £400 must be moved across before the log is taken; logging a sum term by term is not a thing. [4]

SECTION C · Stretch

C1 Predicting inside the fitted range is interpolation and is anchored on both sides. Everything past t = 10 is extrapolation, and its safety falls away with distance. At t = 12 the model has stepped only just outside its window and the growth conditions plausibly still hold, so the prediction is worth having. At t = 100 it claims around 2×10^{14} bacteria, and unbounded exponential growth needs unlimited food and space, which no dish supplies. The mathematics extends for ever. The conditions that justified it do not, and a model's authority stops where its conditions do. B1 for naming extrapolation, B1 for t = 12 sitting just outside the fitted range, B1 for the size of the prediction at t = 100, B1 for the biological limits. An answer that says only “the number is too big” gets one mark of the four; the marks want the fitted range named and the biology brought in. [4]

C2 Set $6 + 1.8 \sin(30t)° > 7$, so $\sin(30t)° > 5/9 = 0.5556$. The principal angle is $30t = 33.75°$, and sine stays above that value until its partner $180° - 33.75° = 146.25°$. So $30t$ runs from $33.75°$ to $146.25°$, giving t from 1.12 to 4.88 hours, a stretch of 3.75 hours in each 12-hour cycle. M1 for the inequality in sine, A1 for 33.75°, M1 for the second angle, A1 for 3.75 hours. The second angle is where the marks are lost: an inequality in sine needs both ends of the window, and a calculator hands over only the first. Sketching one arch of the curve and marking the line $h = 7$ across it settles the direction of the inequality in a moment. [4]