inkMaths

ANSWER BOOK

Further kinematics: acceleration as a function of x, t or v

Further Mechanics 2 · Year FM

6 questions · 22 marks

NAVY EDITION

SECTION A · Warm-up

A1 Use v dv/dx. Writing dv/dt instead leaves an equation containing v, t and x, which cannot be separated because t and x are both unknown functions of each other. Matching the form to the variable the acceleration depends on is what makes the equation separable. B1 for choosing v dv/dx, B1 for why dv/dt fails. [2]

A2 The acceleration depends on t, so dv/dt = 6t - 2 and $v = 3t^2 - 2t + c$. With v = 1 at t = 0, c = 1, so $v = 3t^2 - 2t + 1$. At t = 3: 27 - 6 + 1 = **22** m/s. M1 for integrating, A1 for the expression for v, A1 for 22 m/s. [3]

SECTION B · Standard

B1 The acceleration depends on v and distance is wanted, so use v dv/dx = $-0.02v^2$. Cancelling one v, which is valid while the particle is moving, gives dv/v = -0.02 dx, so ln v = -0.02x + c. With v = 20 at x = 0, $v = 20e^{-0.02x}$. At x = 50: $v = 20e^{-1}$ = **7.36** m/s. M1 for using v dv/dx, M1 for separating and integrating, A1 for $v = 20e^{-0.02x}$, A1 for 7.36 m/s. [4]

B2 Time is wanted and the acceleration depends on v, so dv/dt = -0.4v, giving $v = 15e^{-0.4t}$. Setting v = 7.5: $e^{-0.4t} = 0.5$, so 0.4t = ln2 and t = **1.73** s. M1 for dv/dt = -0.4v, A1 for the expression for v, M1 for setting v = 7.5, A1 for 1.73 s. The same time would halve it again, so the speed never actually reaches zero. [4]

B3 At the terminal speed the acceleration is zero, so $2 = 0.05v^2$ and $v^2 = 40$, giving v = **6.32** m/s. M1 for setting the acceleration to zero, A1 for $v^2 = 40$, A1 for 6.32 m/s. No integration is needed. The terminal speed is simply where the driving effect and the resistance balance, and setting up an integral here wastes most of the time available. [3]

SECTION C · Stretch

C1 The acceleration depends on the displacement, so write it as $v\,dv/dx$. Using dv/dt here leaves three variables in one equation and goes nowhere, and choosing the right form is the first method mark. [6]

Taking the direction of motion as positive, the acceleration is negative: $v\,dv/dx = -6/x^2$.

Separating and integrating: $\int v\,dv = \int -6x^{-2}\,dx$, so $v^2/2 = 6/x + c$.

At $x = 2$, $v = 2$, giving $2 = 3 + c$ and $c = -1$. Hence $\mathbf{v^2} = 12/x - 2$. Find the constant before going any further; carrying it symbolically to the end is where the arithmetic usually breaks.

At $x = 4$: $v^2 = 3 - 2 = 1$, so the speed is **1** m/s.

The particle is furthest from O when it is instantaneously at rest, so set $v = 0$: $12/x = 2$ and $\mathbf{x} = 6$ m. Beyond that point v^2 would be negative, which is the confirmation that 6 m is the limit.

M1 for $v\,dv/dx = -6/x^2$, M1 A1 for integrating, A1 for $c = -1$, A1 for the speed at $x = 4$, A1 for the greatest distance.