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ANSWER BOOK

Impulse and momentum as vectors

Further Mechanics 1 · Year FM

7 questions · 25 marks

NAVY EDITION

SECTION A · Warm-up

A1 Momentum is a vector, so it changes if the direction changes even when the magnitude does not. The impulse equals the vector change mv - mu, which is non-zero whenever v and u point different ways. B1 for momentum being a vector, B1 for the impulse being the vector change. Working with speeds rather than velocities loses that information entirely. [2]

A2 The change in velocity is the impulse divided by the mass: (2i + 3j) m/s. So the new velocity is (2i - 5j) + (2i + 3j) = **(4i** - 2j) m/s, with speed √(16 + 4) = √20 = **4.47** m/s. M1 for dividing the impulse by the mass, A1 for the new velocity, A1 for the speed. [3]

SECTION B · Standard

B1 Impulse = 0.4(-i + 6j) - 0.4(5i + 2j) = 0.4(-6i + 4j) = **(-2.4i** + 1.6j) N s. Its magnitude is √(5.76 + 2.56) = √8.32 = **2.88** N s. M1 for mv - mu, A1 for the impulse, A1 for its magnitude. The speed changed from √29 = 5.39 to √37 = 6.08, so quoting the mass times that difference would have given 0.28 N s: an order of magnitude wrong. [3]

B2 Momenta: 4(2i + 3j) = 8i + 12j and 6(-3i + j) = -18i + 6j. Total = **(-10i** + 18j) N s. Dividing by the combined 10 kg gives v = **(-i** + 1.8j) m/s, with speed √(1 + 3.24) = **2.06** m/s. M1 for the total momentum, A1 for (-10i + 18j), M1 for dividing by the combined mass, A1 for the velocity and speed. [4]

B3 Since |v| = |u|, the vectors mu and mv have the same length, so the triangle they form with the impulse is isosceles. The impulse is therefore perpendicular to the bisector of the angle between the old and new velocities, and in particular it is never along either of them. B1 for the isosceles momentum triangle, B1 for the perpendicular to the bisector, B1 for it lying along neither velocity. [3]

B4 The final velocity is 6i m/s, so the final momentum is 12i and the initial momentum is 2(3i + 4j) = 6i + 8j. The impulse is 12i - (6i + 8j) = **(6i** - 8j) N s, of magnitude √(36 + 64) = **10** N s. M1 for the final momentum, M1 for mv - mu, A1 for the impulse, A1 for its magnitude. The j component had to be removed entirely, so the impulse is not along the direction of travel. [4]

SECTION C · Stretch

C1 Momentum is conserved as a vector, so treat the i and j components separately. [6]

Before: $2(3\mathrm{i} + 4\mathrm{j}) + 2(-\mathrm{i} + 2\mathrm{j}) = (6\mathrm{i} + 8\mathrm{j}) + (-2\mathrm{i} + 4\mathrm{j}) = (\mathbf{4i} + 12\mathrm{j})$ N s.

After: P carries $2(\mathrm{i} + 6\mathrm{j}) = (2\mathrm{i} + 12\mathrm{j})$, so Q carries $(4\mathrm{i} + 12\mathrm{j}) - (2\mathrm{i} + 12\mathrm{j}) = (2\mathrm{i} + 0\mathrm{j})$, giving Q the velocity **i** m/s.

Impulse on P = $2(\mathrm{i} + 6\mathrm{j}) - 2(3\mathrm{i} + 4\mathrm{j}) = (\mathbf{-4i} + 4\mathrm{j})$ N s, of magnitude $\sqrt{32}$ = **5.66** N s.

Impulse on Q = $2(\mathrm{i}) - 2(-\mathrm{i} + 2\mathrm{j}) = (\mathbf{4i} - 4\mathrm{j})$ N s, the same magnitude in the opposite direction, as Newton's third law requires.

M1 for conserving momentum as a vector, A1 for the total before the collision, A1 for the velocity of Q, M1 for an impulse as mv - mu, A1 for the impulse on P, A1 for the impulse on Q.

The two impulses summing to zero is the check worth making, and it says the same thing as conservation of momentum.