inkMaths

ANSWER BOOK

Indices and surds

Algebra and functions · Year 12–13

7 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$. Then $5^0 = 1$ and $4^{-2} = 1/16$. B1 B1 B1, one for each value. [3]
Root before power keeps the numbers small; going the other way means cubing 16 first. A negative index means reciprocal, never a negative answer, and $4^{-2} = -16$ is the slip this question exists to catch.

A2 $\sqrt{72} = \sqrt{(36 \times 2)} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$. M1 for extracting a square factor, A1 for $6\sqrt{2}$. [2]
Pull out the largest square factor in one step. Pulling out 4 first reaches the same place, but takes two rounds and gives the sign of a candidate who has not spotted the 36.

SECTION B · Standard

B1 Work outside in. The minus means reciprocal and the 3 in the denominator means cube root, so $8^{-2/3} = 1/(\sqrt[3]{8})^2 = 1/4$. Likewise $25^{3/2} = (\sqrt{25})^3 = 5^3 = 125$. M1 for treating the negative index as a reciprocal, A1 for $1/4$, A1 for 125. The root and the power belong in the working separately; a bare answer from a calculator answers nothing on a non-calculator paper. [3]

B2 Each surd reduces to a multiple of $\sqrt{2}$. $\sqrt{50} = 5\sqrt{2}$, $\sqrt{32} = 4\sqrt{2}$ and $\sqrt{18} = 3\sqrt{2}$, so the sum is $5\sqrt{2} + 4\sqrt{2} - 3\sqrt{2} = 6\sqrt{2}$. M1 for reducing at least one surd, A1 for all three, A1 for $6\sqrt{2}$. Like terms in $\sqrt{2}$ collect exactly as terms in x would. Adding the numbers under the roots instead gives $\sqrt{64} = 8$, which is wrong and is a favourite distractor. [3]

B3 $5/\sqrt{5} = 5\sqrt{5}/5 = \sqrt{5}$, since multiplying top and bottom by the surd clears it. And $4/\sqrt{6} = 4\sqrt{6}/6 = 2\sqrt{6}/3$. M1 for multiplying top and bottom by the surd, A1 for $\sqrt{5}$, A1 for $2\sqrt{6}/3$. The final cancellation is part of the expected answer, so $4\sqrt{6}/6$ left standing loses the mark. [3]

B4 Multiply top and bottom by the conjugate $\sqrt{6} + 2$. The denominator becomes $6 - 4 = 2$ and the numerator becomes $10(\sqrt{6} + 2)$, so the fraction is $10(\sqrt{6} + 2)/2 = 5\sqrt{6} + 10$, giving $a = 5$ and $b = 10$. M1 for multiplying by the conjugate, A1 for the rational denominator, A1 for $5\sqrt{6} + 10$. The conjugate is chosen precisely so the cross terms cancel and the denominator turns rational. Multiplying by $\sqrt{6} - 2$ instead leaves $10 - 4\sqrt{6}$ downstairs, which is no better than where you started. [3]

SECTION C · Stretch

C1 Rewrite everything over a single base in each equation. The first gives $8^{y+1} = 2^{3(y+1)}$, so $x = 3y + 3$. The second gives $9^y = 3^{2y}$, so $2y = x - 9$. Substituting the first into the second, $2y = 3y + 3 - 9$, so $y = 6$ and then $x = 21$. Checking, $2^{21} = 8^7$ and $9^6 = 3^{12}$, so both hold. M1 for rewriting the first equation over base 2, A1 for $x = 3y + 3$, M1 for rewriting the second over base 3, A1 for $2y = x - 9$, A1 for $y = 6$ and $x = 21$. Equal bases forcing equal exponents is what converts an index problem into simultaneous linear equations, and each conversion carries a mark of its own. Logs would also work here, but they are slower and the numbers stop being exact. [5]