inkMaths

QUESTION WORKBOOK

Induction: divisibility and matrices

Further proof · Year FM

7 questions · 30 marks

PRINT EDITION

SECTION A · Warm-up

A1 Let $f(n) = 8^n - 1$. Show that $f(1)$ is divisible by 7, and write down the inductive hypothesis for the claim that 7 divides $f(n)$ for all positive integers n. [2]

A2 Using $f(n) = 8^n - 1$, simplify $f(k + 1) - 8f(k)$ and hence complete the divisibility proof. [3]

SECTION B · Standard

B1 Prove by induction that $3^{2n} + 7$ is divisible by 8 for all positive integers n. [5]

B2 Let M be the matrix with rows (1, 1) and (0, 1). Prove by induction that M^n has rows (1, n) and (0, 1). [5]

B3 Let D be the matrix with rows (2, 0) and (0, 3). Prove by induction that D^n has rows (2^n, 0) and (0, 3^n). [5]

B4 In divisibility proofs the working often forms f(k + 1) − m f(k) for a chosen multiplier m. Explain how m is chosen, and why the manoeuvre works. [3]

SECTION C · Stretch

C1 M is the matrix with rows (3, -4) and (1, -1). Prove by induction that M^n has rows (1 + 2n, -4n) and (n, 1 - 2n) for all positive integers n. [7]