inkMaths

ANSWER BOOK

Integrating standard functions

Integration · Year 12–13

7 questions · 23 marks

NAVY EDITION

SECTION A · Warm-up

A1 $\int \cos 2x\,dx = \frac{1}{2}\sin 2x + c$, and $\int \sin 4x\,dx = -\frac{1}{4}\cos 4x + c$. B1 B1, one for each integral. Where differentiation multiplied by k, integration divides by it. Sine also picks up a minus sign on the way back, and dropping it is the single commonest error in this topic. [2]

A2 $\int e^{3x}\,dx = e^{3x}/3 + c$, $\int (1/x)\,dx = \ln|x| + c$, and $\int \sec^2 x\,dx = \tan x + c$. B1 B1 B1, one for each integral. Each one is a derivative fact read backwards. The modulus in the logarithm keeps it defined for negative x, where $1/x$ still has perfectly good areas beneath it, and a missing modulus quietly narrows the answer to positive x alone. [3]

SECTION B · Standard

B1 Take the terms one at a time. The integral is $e^{2x}/2 + 3\ln|x| - (\cos 2x)/2 + c$. B1 B1 B1, one for each term. Three standard results, one line each. The danger sits in the last term, where the sine returns a minus sign and the 2 divides at the same time. [3]

B2 $[3\ln x]$ from 1 to $e^2 = 3\ln e^2 - 3\ln 1 = 6 - 0 = 6$. M1 for $3\ln x$, M1 for substituting both limits, A1 for the exact value. The limits are powers of e, so the logarithms evaluate exactly and no calculator is needed. Writing 5.999 throws away the accuracy mark on an exact-value question. [3]

B3 $[\frac{1}{3}\sin 3x]$ from 0 to $\pi/6 = \frac{1}{3}\sin(\pi/2) - 0 = \frac{1}{3}$. M1 for the integrated form, M1 for substituting both limits, A1 for the exact value. The upper limit was chosen so that 3x lands on $\pi/2$, where sine is exactly 1. Set the calculator to radians before checking, because degree mode returns a value near 0.0091 and no marks. [3]

B4 Nothing on the standard list covers $\cos^2 x$, so rewrite it first. The double-angle identity gives $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$. Integrating term by term gives $x/2 + (\sin 2x)/4 + c$. M1 for using the double-angle identity, A1 for $\frac{1}{2} + \frac{1}{2}\cos 2x$, M1 for integrating term by term, A1 for the answer. Identities come before integrals. The algebra turns an impossible integrand into two easy ones, and the alternative of guessing $(\sin^3 x)/3$ or similar is simply wrong. [4]

SECTION C · Stretch

C1 Start from $\cos 4x = 1 - 2\sin^2 2x$, which rearranges to $\sin^2 2x = \frac{1}{2} - \frac{1}{2}\cos 4x$. [5]
Integrating gives $[x/2 - (\sin 4x)/8]$. At the upper limit, $x/2 = \pi/16$ and $\sin 4x = \sin(\pi/2) = 1$, so the bracket is $\pi/16 - 1/8$. At the lower limit it is 0. The value is therefore $\pi/16 - 1/8$, as required. M1 for using $\cos 4x$, A1 for $\frac{1}{2} - \frac{1}{2}\cos 4x$, M1 for integrating, A1 for $x/2 - (\sin 4x)/8$, A1 for reaching the printed answer. Two things have to go right at once. The identity needs the double of the angle already present, so $\sin^2 2x$ calls for $\cos 4x$ rather than $\cos 2x$, and the k-factor of 4 then divides the sine term to give the 8 in the denominator. Candidates who write $\frac{1}{2} - \frac{1}{2}\cos 2x$ reach $\pi/16 - \sqrt{2}/8$, which is wrong and unrecoverable.