inkMaths

ANSWER BOOK

Integration as antidifferentiation

Integration · Year 12–13

7 questions · 23 marks

NAVY EDITION

SECTION A · Warm-up

A1 Reverse the power rule term by term to get $x^3 - x^2 + 5x + c$. M1 for reversing the power rule, A1 for the full answer including the + c. Differentiating that answer lands back on the integrand, a free check worth the ten seconds. Omitting the + c costs a mark on almost every indefinite integral in the paper. [2]

A2 Differentiation destroys constants. Both $x^3 + 7$ and $x^3 - 2$ differentiate to $3x^2$, so running the machine backwards recovers a whole family of curves, one for each constant. The + c holds the family's place until extra information picks a member. B1 for differentiation destroying constants, B1 for the family of curves the + c stands for. [2]

SECTION B · Standard

B1 Rewrite the fraction as $-3x^{-2}$. The integral is then $x^6 + 3x^{-1} + c$, or $x^6 + 3/x + c$. M1 for rewriting as a negative power, A1 for the first term, A1 for the second. Raising -2 by one gives -1, and dividing by -1 flips the sign. That double negative is where the marks are won and lost, and an answer of $x^6 - 3/x + c$ shows it was missed. [3]

B2 Write the integrand as $2x^{1/2} + x^{-1/2}$. Raising each index by one and dividing gives $(4/3)x^{3/2} + 2x^{1/2} + c$. M1 for writing both terms as powers of x, A1 for the first term, A1 for the second. Dividing by 3/2 means multiplying by 2/3, and dividing by 1/2 means doubling. Most of the work here is fraction arithmetic, and most of the lost marks are too. [3]

B3 Integrating gives $y = x^3 - 2x + c$, a family of curves. The point picks the member. Substituting $x = 2$ and $y = 5$ gives $8 - 4 + c = 5$, so $c = 1$ and $y = x^3 - 2x + 1$. M1 for integrating, A1 for the family with its constant, M1 for substituting the point, A1 for the equation of the curve. Stopping at the family, or substituting into dy/dx instead of into y, loses the last two marks. [4]

SECTION C · Stretch

C1 Differentiate the proposal. The chain rule gives $3(x^2 + 1)^2 \times 2x \div 6 = x(x^2 + 1)^2$, which carries an unwanted factor of x. The reversed power rule applies to powers of x, not to powers of a bracket, so the bracket has to be expanded first. Doing so gives $x^4 + 2x^2 + 1$, whose integral is $x^5/5 + 2x^3/3 + x + c$. M1 for differentiating the proposal, A1 for the unwanted factor of x, B1 for the correct integral. [3]

C2 Integrate once to get $dy/dx = 3x^2 - 4x + c$. The gradient condition gives $3 - 4 + c = 5$, so $c = 6$. Integrate again to get $y = x^3 - 2x^2 + 6x + d$. The point condition gives $8 - 8 + 12 + d = 4$, so $d = -8$ and $y = x^3 - 2x^2 + 6x - 8$. M1 for the first integration, A1 for dy/dx with its constant, M1 for using the gradient condition, M1 for the second integration, A1 for y with its second constant, A1 for the final equation. Two integrations mean two constants, and each condition pins exactly one of them. The fatal error is carrying a single c through both integrations, which quietly discards the cx term and makes the final answer unrecoverable. Using the gradient condition first keeps the arithmetic clean, though working the other way round and solving for both constants together is equally valid. [6]