inkMaths

ANSWER BOOK

Kinematics with variable acceleration

Mechanics · Year 12–13

7 questions · 32 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Every suvat equation is derived on the assumption that a is constant, so once a varies those formulae are simply false rather than approximate. In their place, $v = dx/dt$ and $a = dv/dt$, with integration running the chain backwards and the constants of integration fixed by the given starting conditions. B1 for the reason, B1 B1 for the two relationships. [3]

A2 $a = dv/dt = 12t^2 - 6$, so at $t = 2$, $a = 48 - 6 = 42$ m s^{-2}. M1 for differentiating, A1 for the expression, A1 for 42. One differentiation is all that is needed, and the only real care required is not integrating by reflex because the question mentions velocity. [3]

SECTION B · Standard

B1 Maximum velocity occurs where $a = 0$. Since $a = dv/dt = 8 - 4t$, that is at $t = 2$, where $v = 16 - 8 = 8$ m s^{-1}. It is a maximum because a is positive before $t = 2$ and negative after. For the displacement, integrate: $x = 4t^2 - (2/3)t^3 + c$, and the start at the origin gives $c = 0$, so $x(3) = 36 - 18 = 18$ m. M1 for setting $a = 0$, A1 for $t = 2$, A1 for $v = 8$, M1 for integrating with a constant, A1 for 18. Maximum velocity comes from $a = 0$, not from $v = 0$. The two conditions answer completely different questions, and confusing them here gives $t = 4$ and a velocity of zero. [5]

B2 Integrating, $v = 3t^2 - 12t + c$, and $v(0) = 9$ gives $c = 9$. Factorising, $v = 3(t^2 - 4t + 3) = 3(t - 1)(t - 3)$, so the particle is at rest at $t = 1$ and $t = 3$. M1 for integrating with a constant, A1 for v, M1 for solving $v = 0$, A1 for both times. The initial condition has to be used before anything else can be asked of v, and an answer left as +c scores the M mark and nothing beyond it. [4]

B3 Integrating gives $x = t^3 - 6t^2 + 9t$, with no constant since $x(0) = 0$. Then $x(3) = 27 - 54 + 27 = 0$, so the displacement is 0 and the particle is back where it started. Distance is a different matter. Since $v = 3(t - 1)(t - 3)$, the particle changes direction at $t = 1$, where $x(1) = 1 - 6 + 9 = 4$. It travels 4 m forwards from $t = 0$ to $t = 1$ and 4 m backwards from $t = 1$ to $t = 3$, so the total distance is 8 m. M1 A1 for the displacement function, A1 for 0, M1 for finding the turning time, M1 for splitting the journey, A1 for 8 m. Integrating v straight through from 0 to 3 lets the two legs cancel and gives 0 m for the distance. Splitting the journey at the turning time is compulsory rather than optional. [6]

SECTION C · Stretch

C1 Integrate each component separately: $r = (t^3 - 4t)i + 3t^2 j + C$, and putting $t = 0$ gives $C = 2i - j$. So $r = (t^3 - 4t + 2)i + (3t^2 - 1)j$. At $t = 2$, $r = (8 - 8 + 2)i + (12 - 1)j = 2i + 11j$ metres. For the speed, $v(2) = (12 - 4)i + 12j = 8i + 12j$, so the speed is $\sqrt{(64 + 144)} = \sqrt{208} = 4\sqrt{13} = 14.4$ m s^{-1} to 3 significant figures. M1 for integrating, A1 for the components, M1 for using the initial position, A1 for $2i + 11j$, M1 for the magnitude, A1 for 14.4. The constant of integration is a vector, so it needs both components. Speed is the magnitude of velocity, so the answer is a scalar with no i or j in it. [6]

C2 $v = dx/dt = 3t^2 - 18t + 24 = 3(t - 2)(t - 4)$, so the particle is at rest at $t = 2$ and $t = 4$. For the second part, $x = t(t^2 - 9t + 24)$, so $x = 0$ requires either $t = 0$ or $t^2 - 9t + 24 = 0$. That quadratic has discriminant $81 - 96 = -15$, which is negative, so it has no real roots and $t = 0$ is the only time at which $x = 0$. M1 for differentiating, A1 for the factorised velocity, A1 for both times, M1 for factorising out t and testing the quadratic, A1 for the conclusion. Solving $x = 0$ numerically and reporting 'no solutions found' proves nothing. The discriminant is what turns a search into a proof. [5]