inkMaths

ANSWER BOOK

Linear programming: formulation and graphical solution

Decision Mathematics 1 · Year FM

7 questions · 26 marks

NAVY EDITION

SECTION A · Warm-up

A1 The variables defined in words with their units, the objective to be maximised or minimised, and the constraints as inequalities. B1 for the three components of the formulation, B1 for naming the usual omission. The non-negativity conditions are the commonest omission; they bound the region and carry a mark of their own. [2]

A2 $3x + 2y + s_1 = 20$, where s_1 is a **slack** variable taking up the unused amount. $5x + 2y - s_2 + t_1 = 30$, where s_2 is a **surplus** recording the excess over 30 and t_1 is an **artificial** variable providing a starting solution. B1 for the slack equation, B1 for the surplus, B1 for the artificial variable. [3]

SECTION B · Standard

B1 Let **x** be the number of tables made in a week and **y** the number of chairs. [4]
Both must be defined in words; a formulation that opens with an inequality loses the first mark.
Maximise **P** = $30x + 16y$ (profit in pounds).
Machining: **4x** + $y \le 40$.
Finishing: $2x + 2y \le 36$, which simplifies to **x** + $y \le 18$.
And **x** ≥ 0, $y \ge 0$. B1 for defining the variables in words, B1 for the objective, B1 for the two resource constraints, B1 for the non-negativity conditions. Keep the units straight throughout: hours on the left of the resource constraints, pounds in the objective.

B2 The vertices are $(0, 0)$, $(10, 0)$, $(0, 8)$ and the intersection of the two lines. [4]
Solving $x + y = 10$ with $2x + 3y = 24$: $2x + 3(10 - x) = 24$, so $30 - x = 24$ and $x = 6$, $y = 4$.
Evaluating P: 0, 40, 40 and **44**. The maximum is **44** at $(6, 4)$. M1 for listing the vertices, M1 for solving the two lines simultaneously, A1 for the intersection, A1 for the maximum. Test every vertex and show the values; naming the corner without them scores the answer mark only.

B3 Draw one line of constant objective value, say $4x + 5y = 20$, and slide it parallel away from the origin. The last corner it touches before leaving the region is the optimum, here **(6,** 4). It also shows at once when the objective is parallel to a constraint, in which case a whole edge is optimal and there are infinitely many solutions. Testing the vertices would report that only as a tie between two corners. B1 for drawing a line of constant objective value, B1 for sliding it to the last point of the region, B1 for the parallel-edge case. [3]

B4 At $(6, 4)$: $6 + 4 + s_1 = 10$ gives $s_1 =$ **0**, and $12 + 12 + s_2 = 24$ gives $s_2 =$ **0**. Both constraints are binding, so there is no spare capacity in either. M1 for substituting into the constraints, A1 for both slacks being zero, B1 for the interpretation. Relaxing either one on its own would allow a larger P. [3]

SECTION C · Stretch

C1 The vertices are (0, 0), (2.5, 0), (0, 4) and the intersection of the two constraint lines. From $4x + y = 10$, $y = 10 - 4x$; substituting into $2x + 3y = 12$ gives $2x + 30 - 12x = 12$, so $x =$ **1.8** and $y =$ **2.8**. [7]

Values of P: 0, 12.5, 16 and $5(1.8) + 4(2.8) = 9 + 11.2 =$ **20.2**. The maximum is 20.2 at (1.8, 2.8).

Rounding up to (2, 3) gives $4(2) + 3 = 11$, which breaks $4x + y \le 10$, so that point is **outside** the feasible region and cannot be used at all. Rounding down to (1, 2) is feasible but gives only P = **13**.

Test the lattice points of the region instead. Those satisfying both constraints include (0, 4) with P = 16, (1, 3) with P = 17 and (2, 2) with P = **18**, which is the largest. Note that $4(2) + 2 = 10$ and $2(2) + 3(2) = 10$, so (2, 2) sits on one constraint line and inside the other. The integer optimum is **P** = 18 at (2, 2).

M1 for the vertices, M1 for solving the two lines, A1 for the intersection, A1 for the continuous maximum, M1 for testing lattice points, A1 for the integer optimum, B1 for why rounding fails.

Neither rounding found it, because (2, 2) is not a neighbour of (1.8, 2.8) in the y direction at all. The integer optimum can sit some way from the continuous one. Search the region; do not round the corner.