inkMaths

ANSWER BOOK

Log graphs and exponential models

Exponentials and logarithms · Year 12

7 questions · 25 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Plot log y against log x. Taking logs of $y = ax^n$ gives $\log y = \log a + n \log x$, so the points fall on a straight line with gradient n and intercept log a. Straightness on logged axes is the evidence for the law. B1 for what to plot, B1 for the gradient, B1 for the intercept. The intercept is log a, not a, and reporting it as a is where this loses a mark. [3]

A2 Plot log y against x itself rather than log x. The law gives $\log y = \log a + x \log b$, a line with gradient log b and intercept log a. B1 for what to plot, B1 for the gradient, B1 for the intercept. Which axis gets logged is the only difference between the two tests, and it is decided by where the variable sits. In $y = ab^x$ the x is an exponent, so it is already the thing a log would produce and needs no logging. [3]

SECTION B · Standard

B1 The log-log gradient is $n = (\log 135 - \log 40)/(\log 9 - \log 4) = \log 3.375 \div \log 2.25 = 1.5$ exactly, since $3.375 = 2.25^{1.5}$. Then $a = 40/4^{1.5} = 40/8 = 5$, and the law is $y = 5x^{1.5}$. M1 for forming the log-log gradient, A1 for n = 1.5, M1 for substituting a point to find a, A1 for the law. The second point checks it: $5 \times 9^{1.5} = 5 \times 27 = 135$. [4]

B2 The initial mass is the front constant, 60 g, since $e^0 = 1$. After 5 days, $m = 60e^{-0.5} = 36.4$ g. Halving means $e^{-0.1t} = ½$, so $-0.1t = \ln ½$ and $t = \ln 2/0.1 = 6.93$ days. B1 for 60 g, M1 for substituting t = 5, A1 for 36.4 g, M1 for setting the exponential to ½ and taking logs, A1 for 6.93 days. The half-life never depends on the starting mass, so the 60 cancels before the log is taken; carrying it through to write ln 30 is the usual wrong turn. [5]

B3 Plot log y against x. Taking logs: $\log y = \log 3 + x \log 5$, so the gradient is $\log 5 = 0.699$ and the intercept is $\log 3 = 0.477$. B1 for what to plot, B1 for 0.699, B1 for 0.477. The growth factor lives in the gradient, the starting value in the intercept. [3]

SECTION C · Stretch

C1 The line says $\log y = 0.5 \log x + \log 6$, so the law is $y = 6x^{0.5} = 6\sqrt{x}$. At x = 25, $y = 6 \times 5 = 30$. M1 for undoing the logs, A1 for the law, M1 for substituting x = 25, A1 for 30. Undoing the logs turns the intercept back into the front constant and the gradient back into the power. [4]

C2 $m(t + 7)/m(t) = e^{-0.1(t+7)}/e^{-0.1t} = e^{-0.7} = 0.497$, with the t-dependence cancelling entirely. M1 for forming the ratio, A1 for the t-dependence cancelling, A1 for 0.497. Equal time steps multiply the mass by equal factors: the signature no other kind of curve shares. [3]