

inkMaths

ANSWER BOOK

Maclaurin series

Further algebra and series · Year FM

7 questions · 27 marks

NAVY EDITION

SECTION A · Warm-up

- A1** $f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$: the coefficient of x^r is the r th derivative at zero divided by r factorial. B1 for the expansion, B1 for the general coefficient. [2]
- A2** The derivatives at 0 cycle 1, 0, -1, 0, 1, so $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots$. M1 for the derivative values at 0, A1 for the x^2 term, A1 for the x^4 term. Only even powers survive, matching $\cos$ being an even function. [3]

SECTION B · Standard

- B1** $1 - 0.02 + 0.0000667 = 0.980067$, and the true value is 0.980067 to six decimal places. The next term is $0.2^6/720$, about 9×10^{-8} . M1 for substituting into the three terms, A1 for 0.980067, B1 for the comment on accuracy. Three terms are tight here only because 0.2 is small; at $x = 2$ the same three terms give -0.333, against a true value of -0.416. [3]
- B2** Substitute $3x$ into the exponential series: $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{6} = 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + \dots$. The x^3 coefficient is $27/3! = 9/2$. M1 for substituting $3x$, A1 for the x^2 term, A1 for the x^3 term, A1 for the coefficient $9/2$. Substitution into a standard series beats fresh differentiation every time it is available. [4]
- B3** $\ln(1+x)$ converges for $-1 < x \leq 1$. Substituting $3x$ means the substituted quantity must stay in that window: $-1 < 3x \leq 1$, so $-1/3 < x \leq 1/3$. B1 for the original window, M1 for substituting $3x$, A1 for the new window. The window transforms with the same substitution as the series. [3]

SECTION C · Stretch

- C1** Subtracting the two log series cancels the even powers and doubles the odd ones: $2(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots)$. At $x = 1/3$ the argument is $(4/3)/(2/3) = 2$, and two terms give $2(1/3 + 1/81) = 0.6914$, against $\ln 2 = 0.6931$. M1 for subtracting the two series, A1 for the even powers cancelling, A1 for the odd-power form, M1 for putting $x = 1/3$, A1 for the argument being 2, A1 for 0.6914. Far faster than the plain $\ln(1+x)$ series at $x = 1$, which needs hundreds of terms for the same accuracy. [6]

- C2** Two standard series stack here. Write $u = \sin x = x - x^3/6 + \dots$, then $\ln(1 + u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \dots$. Work to x^3 throughout, so $u^2 = x^2 + \dots$ with the next correction at x^4 , and $u^3 = x^3 + \dots$. Substituting, $\ln(1 + \sin x) = (x - x^3/6) - x^2/2 + x^3/3 + \dots = x - x^2/2 + x^3/6 + \dots$. M1 for putting $u = \sin x$, M1 for the $\ln(1 + u)$ expansion, A1 for u^2 to the required order, A1 for u^3 , M1 for substituting and collecting, A1 for the final series. Differentiating four times instead is legitimate but long; the second derivative of $\ln(1 + \sin x)$ alone is a quotient with $\sin x$ and $\cos^2 x$ in it. Decide how far you need each series before substituting, or the x^3 coefficient picks up terms that should have been discarded.
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