



ANSWER BOOK

Matrix algebra and transformations

Matrices · Year FM

7 questions · 27 marks

NAVY EDITION

SECTION A · Warm-up

- A1** Rotation: rows $(0, -1)$ and $(1, 0)$. Reflection in $y = x$: rows $(0, 1)$ and $(1, 0)$. B1 B1 for the two matrices. Both can be read off from where $(1, 0)$ and $(0, 1)$ land, since the images are the columns. [2]
- A2** AB has rows $(4, -2)$, $(6, 0)$; BA has rows $(2, -2)$, $(4, 2)$. They differ, because matrix multiplication is not commutative, so the order of a product must never be swapped mid-working. M1 for a correct multiplication method, A1 for both products, B1 for the comment on commutativity. [3]

SECTION B · Standard

- B1** Multiply: rows $(0, -1)$, $(1, 0)$ applied to $(3, 1)$ give $(-1, 3)$. B1 for the rotation matrix, M1 for the multiplication, A1 for $(-1, 3)$. A sketch confirms it. The point swings from the first quadrant into the second, a quarter turn against the clock. [3]
- B2** The reflection acts first, so it sits on the right of the product. RM is rows $(0, -1)$, $(1, 0)$ times rows $(1, 0)$, $(0, -1)$, which gives rows $(0, 1)$, $(1, 0)$. That is the reflection in $y = x$. B1 for the reflection matrix, M1 for multiplying in the order RM, A1 for rows $(0, 1)$, $(1, 0)$, A1 for naming the reflection in $y = x$. Writing MR instead gives rows $(0, -1)$, $(-1, 0)$, the reflection in $y = -x$, so the order genuinely matters. [4]
- B3** The images of the base vectors are the columns, giving rows $(2, 1)$, $(0, 3)$. Then $(2, 5)$ maps to $(2 \times 2 + 1 \times 5, 0 + 3 \times 5) = (9, 15)$. B1 for the matrix, M1 for the multiplication, A1 for $(9, 15)$. Columns-are-images turns matrix building into reading the question. [3]
- B4** Squaring gives rows $(-1, 0)$, $(0, -1)$, which is $-I$, a half turn. Squaring again gives $(-1)^2 = I$, so $R^4 = I$. M1 for squaring, A1 for $-I$, M1 for squaring again, A1 for the fourth power being the identity. Four quarter turns make a full turn, and the matrix records that as its fourth power being the identity. [4]

SECTION C · Stretch

- C1** Build each matrix from the images of the base vectors. R fixes i , sends j to k and k to $-j$, so R has rows $(1, 0, 0)$, $(0, 0, -1)$, $(0, 1, 0)$. S fixes k , sends i to j and j to $-i$, so S has rows $(0, -1, 0)$, $(1, 0, 0)$, $(0, 0, 1)$. S first means S on the right, so the combined matrix is RS, with rows $(0, -1, 0)$, $(0, 0, -1)$, $(1, 0, 0)$. Applying it to $(1, 2, 3)$ gives $(-2, -3, 1)$. M1 for building a matrix from the images of the base vectors, A1 for R, A1 for S, M1 for the product in the order RS, A1 for RS, A1 for $(-2, -3, 1)$, M1 for reversing the order, A1 for SR differing. Reversing the order gives SR, with rows $(0, 0, 1)$, $(1, 0, 0)$, $(0, 1, 0)$, which sends $(1, 2, 3)$ to $(3, 1, 2)$. The two matrices differ, so the transformations differ. Rotations about different axes generally do not commute, and these two 90° rotations give different products, which is easy to confirm with a die or a book held at arm's length. [8]
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