inkMaths

ANSWER BOOK

Mixed strategies

Decision Mathematics 2 · Year FM

6 questions · 26 marks

NAVY EDITION

SECTION A · Warm-up

A1 If maximin and minimax differ, then whichever single pair of choices is made, one player can improve by switching. A predictable choice can be exploited, so the row player has to vary between rows with fixed probabilities. B1 for the instability, B1 for the need to randomise. Being unpredictable in the right proportion is what raises the guaranteed pay-off above the maximin. [2]

A2 Equate the two: $2 + 5p = 7 - 3p$, so $8p = 5$ and $p =$ **0.625**. The value is $2 + 5(0.625) =$ **5.125**, and $7 - 3(0.625) = 5.125$ confirms it. M1 for equating the two expressions, A1 for $p = 0.625$, A1 for the value 5.125. [3]

SECTION B · Standard

B1 Row minima are 2 and 3, so the row player's maximin is 3. Column maxima are 6 and 7, so the column player's minimax is 6. They differ, so there is **no** stable solution and no saddle point. [6]
Let the row player play R1 with probability p, so R2 has probability $1 - p$. Against C1 the expected pay-off is $6p + 3(1 - p) =$ **3** $+ 3p$. Against C2 it is $2p + 7(1 - p) =$ **7** $- 5p$.
Equating: $3 + 3p = 7 - 5p$, so $8p = 4$ and $p =$ **0.5**.
The value is $3 + 3(0.5) =$ **4.5**, and $7 - 5(0.5) = 4.5$ agrees. B1 for the maximin 3, B1 for the minimax 6, B1 for stating there is no stable solution, M1 for the two expected pay-offs, A1 for $p = 0.5$, A1 for the value 4.5. The row player plays each row half the time, and 4.5 lies between the maximin of 3 and the minimax of 6 as it must.

B2 Let the column player play C1 with probability q. Against R1 the expected loss is $6q + 2(1 - q) =$ **2** $+ 4q$. Against R2 it is $3q + 7(1 - q) =$ **7** $- 4q$. [5]
Equating: $2 + 4q = 7 - 4q$, so $8q = 5$ and $q =$ **0.625**.
The value is $2 + 4(0.625) =$ **4.5**, matching the value the row player's mix gives, so both mixes are right. M1 for the two expected losses, A1 for both expressions, M1 for equating them, A1 for $q = 0.625$, A1 for the value 4.5. Note that the two probabilities differ even though the value does not.

B3 The proportions are fixed by where the two expected pay-off lines cross, not by the sizes of the entries. Here the answer is an even split, even though R2 has the larger total. Favouring R2 would let the column player reply with C1, where R2 pays only 3, and that drags the guaranteed pay-off below 4.5. The value of a mixed strategy is set by the worst reply, never by the average of a row. B1 for the proportions coming from the crossing point, B1 for the column player's reply, B1 for the effect on the guaranteed pay-off. [3]

SECTION C · Stretch

C1 With R1 played with probability p, the expected pay-offs are $2 + 2p$ against C1, $7 - 6p$ against C2, and $3 + 3p$ against C3. Sketch all three over 0 to 1 and take the highest point of the lower boundary. [7]

At $p = 0$ the values are 2, 7 and 3; at $p = 1$ they are 4, 1 and 6. The lower boundary is C1 rising and C2 falling, and they meet where $2 + 2p = 7 - 6p$, so $8p = 5$ and $p =$ **0.625**, giving a value of **3.25**.

There C3 is worth $3 + 3(0.625) = 4.875$, above the boundary, so **C3** is never used: it would hand the row player more than 3.25.

M1 for the three expected pay-off expressions, A1 for all three correct, M1 for reading the lower boundary, A1 for $p = 0.625$, A1 for the value 3.25, B1 for the column never used, B1 for the linear programming route.

A 3 by 3 game has no single p to plot, so it is converted into a **linear** program: add a constant to every entry to make them positive if needed, maximise the value subject to one constraint per opposing choice, solve by Simplex, then subtract the constant again.