



QUESTION WORKBOOK

Newton-Raphson and the trapezium rule

Numerical methods · Year 13

7 questions · 23 marks

PRINT EDITION

SECTION A · Warm-up

A1 Write down the Newton-Raphson iteration formula for solving $f(x) = 0$. [2]

A2 The trapezium rule with n strips of width h estimates an integral from the ordinates $y_0, y_1, \dots, y_n$. Write down the rule. [2]

SECTION B · Standard

B1 $f(x) = x^3 + x - 5$. Using the Newton-Raphson method with $x_0 = 1.5$, find x_1 and x_2 , giving x_2 to 5 decimal places. [4]

B2 Use the trapezium rule with 4 strips to estimate $\int_1^2 \ln x \, dx$, giving your answer to 4 decimal places. [4]

- B3** The exact value of $\int_1^2 \ln x \, dx$ is $2 \ln 2 - 1 = 0.3863$. Explain why a trapezium-rule estimate of this integral comes out too small. [2]

SECTION C · Stretch

- C1** $f(x) = x^3 - 3x + 1$. Explain why the Newton-Raphson method cannot be applied with $x_0 = 1$. Taking $x_0 = 0.9$ instead, find x_1 and comment on the result, given that $f(x) = 0$ has roots near -1.879 , 0.347 and 1.532 . [6]

- C2** State one way to improve a trapezium-rule estimate of $\int_1^2 \ln x \, dx$, and state one check that the improved estimate is behaving as expected. [3]
