inkMaths

ANSWER BOOK

Oblique impact and impact with a smooth surface

Further Mechanics 1 · Year FM

7 questions · 27 marks

NAVY EDITION

SECTION A · Warm-up

A1 The component parallel to the surface is unchanged, because a smooth surface exerts no force along itself. The component perpendicular to the surface is reversed and multiplied by e, exactly as in a direct impact, because the impulse acts along that direction only. B1 B1 for the two components, each with its reason. [2]

A2 Parallel component = 12cos40° = **9.19** m/s, unchanged. Perpendicular component = 12sin40° = 7.71 m/s, becoming 0.6 × 7.71 = **4.63** m/s the other way. Speed = $\sqrt{(9.19^2 + 4.63^2)}$ = $\sqrt{105.9}$ = **10.3** m/s. M1 for resolving and applying e to the perpendicular component, A1 for 4.63 m/s, A1 for the resultant speed. [3]

SECTION B · Standard

B1 The components afterwards are 9.19 m/s along the wall and 0.6 × 12sin40° = 4.63 m/s away from it, so the angle is arctan(4.63/9.19) = **26.7°**, shallower than the 40° it arrived at. [4]

For the energy, work with squared speeds and let the mass cancel: $9.19^2 + 4.63^2 = 105.9$ remains out of $12^2 = 144$. The fraction lost is 1 - 105.9/144 = 0.264, that is **26.4%**. M1 A1 for the angle, M1 A1 for the percentage lost. The mass never has to be known, and putting in a value for it wastes time.

B2 Let the parallel component be p, unchanged by the impact. Before, the perpendicular component is p tan45°; after, it is p tan30°. Restitution applies to that component alone, so e = p tan30°/(p tan45°) = tan30°/tan45° = **0.577** to three decimal places. M1 for the parallel component unchanged, M1 for restitution applied to the perpendicular components, A1 for tan30°/tan45°, A1 for 0.577. The angles alone settle it, and neither speed is needed. [4]

B3 The tangent of the angle to the surface is the perpendicular component divided by the parallel one. The impact multiplies the numerator by e and leaves the denominator alone, so the tangent is multiplied by e. For e < 1 that makes the angle smaller. M1 for the tangent as the ratio of the two components, A1 for the conclusion. Only e = 1 leaves it unchanged, the mirror case. [2]

B4 Take the two walls along the axes, so the velocity is (u, v). The first wall reverses and scales the component perpendicular to it, giving (-eu, v), and leaves the other component alone. [4]
The second wall does the same to the other component, giving **(-eu,** -ev) = -e(u, v).
That is a negative scalar multiple of the original velocity, so the direction is exactly reversed, and the speed is e times the original. M1 A1 for the effect of the first wall, M1 A1 for the second and the conclusion. It is why a ball played into the corner of a court comes straight back.

SECTION C · Stretch

C1 Resolve A's velocity along and perpendicular to the **line** of centres. That line, not the direction of motion, is where the impulse acts. [8]
Along: 6cos30° = **3√3** ≈ 5.196 m/s. Perpendicular: 6sin30° = **3** m/s.
The spheres are smooth, so the perpendicular component of A is unchanged and B gains none. Everything else happens along the line of centres.
Taking unit mass, conservation of momentum along that line gives $v_A + v_B = 3\sqrt{3}$, and the law of restitution gives $v_B - v_A = 0.5(3\sqrt{3}) = 1.5\sqrt{3}$.
Adding and halving: v_B = **2.25√3** ≈ 3.90 m/s along the line of centres. Subtracting: v_A = **0.75√3** ≈ 1.30 m/s along it.
B moves along the line of centres at 3.90 m/s, since it had no perpendicular component to keep.
A has components 1.30 along and 3 perpendicular, so its speed is √(1.6875 + 9) = √10.6875 = **3.27** m/s, at arctan(3/1.30) = **66.6°** to the line of centres. A is deflected much further from that line than it arrived.
Kinetic energy, per unit mass and dropping the common ½: before, $6^2 = 36$. After, 10.6875 + 15.1875 = 25.875, taking B's contribution as $(2.25\sqrt{3})^2 = 15.1875$.
The fraction lost is (36 - 25.875)/36 = 10.125/36 = **9/32**, that is 0.28125 or **28.1%**. M1 A1 for resolving along and perpendicular to the line of centres, M1 for conservation of momentum, M1 for the restitution equation, A1 A1 for the two speeds along that line, A1 for A's speed and direction, A1 for the fraction lost. Working in squared speeds throughout means the mass and the ½ never appear.