inkMaths

ANSWER BOOK

Oscillations on strings and springs

Further Mechanics 2 · Year FM

7 questions · 27 marks

NAVY EDITION

SECTION A · Warm-up

A1 At equilibrium the tension already balances the weight. Measuring the displacement from there, the extra tension is $\lambda x/l$ and the weight has cancelled out, leaving a restoring force proportional to x. The weight therefore fixes where the oscillation is centred but has no effect on the motion about that point. B1 B1 for the weight cancelling and for the restoring force proportional to x. [2]

A2 $\omega^2 = \lambda/(ml) = 40/(1 \times 0.5) =$ **80**, so $\omega =$ **8.94** rad/s and $T = 2\pi/\omega =$ **0.702** s. M1 for $\omega^2 = \lambda/(ml)$, A1 for ω, A1 for the period. [3]

SECTION B · Standard

B1 At equilibrium the tension equals the weight, so $40e/0.5 = 1(9.8)$. Then $80e = 9.8$ and $e =$ **0.1225** m. M1 for equating tension to weight, A1 for $80e = 9.8$, A1 for the extension. Note that $\omega^2 = \lambda/(ml) = g/e$, a useful check here since $9.8/0.1225 = 80$. [3]

B2 Released from rest at 0.1225 m above equilibrium, so the amplitude is **0.1225** m. The greatest speed is $a\omega = 0.1225 \times 8.944 =$ **1.10** m/s, reached at the equilibrium position, where the net force is zero and the acceleration vanishes. B1 for the amplitude, M1 A1 for the greatest speed, B1 for placing it at the equilibrium position. [4]

B3 A string cannot push, so above the natural length it goes slack and exerts nothing. Released from exactly the natural length, the motion is still simple harmonic throughout, since the particle only just reaches that point. Any greater amplitude and the particle would rise above it as a free body under gravity, so the motion would be simple harmonic for part of each cycle only. B1 for the string going slack above the natural length, B1 for the motion staying simple harmonic in this case, B1 for what a greater amplitude would do. [3]

B4 Displacing the particle by x stretches one spring by an extra x and shortens the other by x. Both then push it back, each with force $25x/0.4$, so the total restoring force is $2 \times 25x/0.4 = 125x$. Hence $0.5\ddot{x} = -125x$, giving $\omega^2 = 2\lambda/(ml) =$ **250** and $\omega = 15.8$ rad/s. $T = 2\pi/15.8 =$ **0.397** s. M1 for the total restoring force, A1 for $\omega^2 = 250$, M1 for $T = 2\pi/\omega$, A1 for 0.397 s. Two springs make the system stiffer, and the period shortens by a factor of $\sqrt{2}$. Both springs push the particle back, so the forces add rather than cancel, and that is the step most often got wrong. [4]

SECTION C · Stretch

C1 At equilibrium $24.5e/1 = 0.5(9.8)$, so $e =$ **0.2** m, and $\omega^2 = \lambda/(ml) = 24.5/0.5 = 49$, giving $\omega =$ **7** rad/s. [8]

While the string is taut the motion is simple harmonic about the equilibrium position with amplitude $a = 0.5$ m. The string goes slack when the particle is level with the natural length, that is at displacement $x = 0.2$ m above the centre. Then $v^2 = \omega^2(a^2 - x^2) = 49(0.25 - 0.04) = 10.29$, so $v =$ **3.21** m/s upwards.

Above that point the string exerts nothing, so the particle is a free body under gravity alone and the motion is no longer simple harmonic. It rises a further $v^2/(2g) = 10.29/19.6 =$ **0.525** m.

Greatest height above the equilibrium position = 0.2 + 0.525 = **0.725** m.

M1 A1 for the equilibrium extension, B1 for $\omega = 7$, M1 for locating where the string goes slack, M1 A1 for the speed there, M1 A1 for the further rise and the total height. The amplitude 0.5 m exceeds the extension 0.2 m, so the string must go slack, and using $v^2 = \omega^2(a^2 - x^2)$ all the way to the top would be wrong. Marks are awarded for saying where the simple harmonic motion stops.