inkMaths

ANSWER BOOK

Parametric equations

Coordinate geometry · Year 12–13

7 questions · 23 marks

NAVY EDITION

SECTION A · Warm-up

A1 $x = 4 + 1 = 5$ and $y = 4 - 1 = 3$, so the point is $(5, 3)$. One value of the parameter gives one point of the curve. Both coordinates come from the same t, so a mark goes to each substitution. B1 B1, one for each coordinate. [2]

A2 The simpler equation gives $t = x + 3$, and substituting into the other gives $y = (x + 3)^2$. M1 for $t = x + 3$ substituted, A1 for the Cartesian equation. Solve the easier equation for t and feed the harder one; that route handles almost every algebraic pair. The answer must be free of t, so any t left standing means the conversion is unfinished, however tidy the algebra. [2]

SECTION B · Standard

B1 $t = x/2$, so $y = 12 \div (x/2) = 24/x$, which rearranges to $xy = 24$. That is a hyperbola with the axes as asymptotes. M1 for eliminating t, A1 for $xy = 24$, B1 for naming the hyperbola. The excluded value $t = 0$ lines up with the curve never reaching $x = 0$, so nothing extra needs removing. Dividing by a fraction means multiplying by its reciprocal, and answering $y = 6/x$ is the slip that follows from forgetting it. [3]

B2 Squaring and adding: $x^2 + y^2 = 25(\cos^2 t + \sin^2 t) = 25$, the circle of radius 5 about the origin. On $0 \le t \le \pi$ the value $y = 5 \sin t$ is never negative, so C is the upper semicircle only, running from $(5, 0)$ at $t = 0$ to $(-5, 0)$ at $t = \pi$. M1 for squaring and adding, A1 for the circle, B1 for the upper semicircle, B1 for the two endpoints. Trig pairs convert by identity, and the parameter's range then says how much of the curve is really there. [4]

B3 $\cos t = x/4$ and $\sin t = y/3$, so the identity $\sin^2 t + \cos^2 t = 1$ gives $x^2/16 + y^2/9 = 1$. That is an ellipse, 4 wide and 3 tall measured from the centre. M1 for $\cos t = x/4$ and $\sin t = y/3$, dM1 for substituting into the identity, A1 for the ellipse. Divide by the coefficients before squaring; squaring first buries them and the identity no longer fits. Note the denominators are 16 and 9, the squares of 4 and 3. [3]

SECTION C · Stretch

C1 $y = (t^2)^3$, so $y = x^3$. But $x = t^2 \ge 0$ for every real t, so C is only the half of the cubic with $x \ge 0$. M1 for cubing $x = t^2$, A1 for $y = x^3$, B1 for the restriction $x \ge 0$. The Cartesian equation says where a point may be; the parametrisation says where the point actually goes. [3]

C2 Landing means $y = 0$, and $15t - 5t^2 = 5t(3 - t)$ gives $t = 0$ at launch or $t = 3$. So the ball lands after 3 seconds, at $x = 20 \times 3 = 60$ metres. The greatest height sits at the vertex of the y parabola, halfway between the two roots at $t = 1.5$, where $y = 22.5 - 11.25 = 11.25$ metres. Differentiating y with respect to t and setting $dy/dt = 15 - 10t = 0$ reaches the same time. M1 for setting $y = 0$, A1 for $t = 3$, B1 for 60 metres, M1 for the vertex of the parabola, A1 for $t = 1.5$, A1 for 11.25 metres. One clock drives both coordinates, so solve the vertical story for time and then hand that time to the horizontal one. Discarding $t = 0$ without saying it is the launch, or quoting the greatest height as an x value, are the two ways this loses marks. [6]