inkMaths

ANSWER BOOK

Probability and Venn diagrams

Statistics · Year 12–13

6 questions · 22 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Mutually exclusive means the events cannot happen together, so $P(A \cap B) = 0$. [2]
The addition rule then loses its correction term and $P(A \cup B) = 0.45 + 0.35 = 0.8$. B1 for $P(A \cap B) = 0$, B1 for $P(A \cup B) = 0.8$. On a Venn diagram the two circles are drawn apart, with no region between them.

A2 Rearrange the addition rule: $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.55 + 0.4 - 0.75 = 0.2$. Now test independence. $P(A) \times P(B) = 0.55 \times 0.4 = 0.22$, which does not equal 0.2, so A and B are not independent. M1 for rearranging the addition rule, A1 for 0.2, B1 for the independence test with both values quoted. The conclusion must state the comparison and not only the verdict, so quote both 0.22 and 0.2. The same test in conditional form gives $P(A|B) = 0.2/0.4 = 0.5$ against $P(A) = 0.55$, and reaches the same answer. [3]

SECTION B · Standard

B1 Put the overlap in first: 20 study both. Art only is then 55 - 20 = 35 and music only is 40 - 20 = 20. Those three regions hold 75, so 25 students sit outside both circles. B1 for the overlap, M1 for subtracting it, A1 for 35 and 20, B1 for the 25 outside. Filling from the middle outwards is the whole technique. The figures 55 and 40 already include the 20, so subtracting the overlap before anything else stops it being counted twice. [4]

B2 Exactly one means art only or music only, so $(35 + 20)/100 = 0.55$. Neither is the region outside both circles, $25/100 = 0.25$. M1 for adding the two single regions, A1 for 0.55, B1 for 0.25. 'Exactly one' deliberately excludes the overlap, which is what makes it a different question from $P(A \cup B) = 0.75$. Read the wording before reaching for a region. [3]

B3 Mutually exclusive events cannot both happen, so their circles do not overlap and $P(A \cap B) = 0$. Independent events are those where one happening tells you nothing about the other, so $P(A \cap B) = P(A)P(B)$. B1 for mutual exclusivity, B1 for independence, B1 for why they cannot hold together. The two ideas are distinct properties rather than opposites. For events of non-zero probability they cannot hold together, because mutual exclusivity is an extreme form of dependence, with knowing that A occurred ruling B out entirely — and a pair of events may easily be neither exclusive nor independent. [3]

SECTION C · Stretch

C1 Fill the three-circle diagram from the centre outwards. The middle holds 5. Tea and coffee but not juice is 12 - 5 = 7; coffee and juice but not tea is 10 - 5 = 5; tea and juice but not coffee is 8 - 5 = 3. Now the single regions. Tea only is 30 - 7 - 3 - 5 = 15, coffee only is 25 - 7 - 5 - 5 = 8, and juice only is 22 - 3 - 5 - 5 = 9. Those seven regions total 52, so 60 - 52 = 8 people like none of the three. Exactly one drink covers 15 + 8 + 9 = 32 people, a probability of 32/60 = 8/15, or 0.533. The inclusion-exclusion formula gives the same total: 30 + 25 + 22 - 12 - 10 - 8 + 5 = 52. B1 for the centre, M1 A1 for the three pairwise regions, A1 for the three single regions, A1 for the 8 who like none, M1 A1 for the probability. Order is everything here. Subtracting the triple overlap from each pair before touching the single regions is what keeps every person counted exactly once, and working outwards in the other direction always double counts. [7]