InkMaths

QUESTION WORKBOOK

Probability generating functions

Further Statistics 1 · Year FM

6 questions · 25 marks

PRINT EDITION

inkphysics.co.uk/maths

SECTION A · Warm-up

A1 Define the probability generating function of a discrete random variable X, and state the value of G(1). [2]

A2 X has $G(t) = (0.6 + 0.4t)^8$. Name the distribution and find E(X). [3]

SECTION B · Standard

B1 X has $G(t) = (0.6 + 0.4t)^8$, with E(X) = 3.2. Find Var(X) using the generating function. [4]

B2 Derive the probability generating function of $X \sim Po(\lambda)$, and use it to confirm $E(X) = \lambda$. [5]

B3 X and Y are independent with $X \sim Po(2)$ and $Y \sim Po(7)$. Use generating functions to identify the distribution of $X + Y$. [3]

SECTION C · Stretch

C1 A geometric distribution has $G(t) = pt/(1 - qt)$ with $q = 1 - p$. Show that its variance is q/p^2. The number of trials up to and including the third success is the sum of three independent such variables. Write down its probability generating function and deduce its mean and variance. [8]