InkMaths

QUESTION WORKBOOK

Proof by induction: sums and series

Further proof · Year FM

7 questions · 26 marks

PRINT EDITION

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SECTION A · Warm-up

A1 State the two things a proof by induction must establish, and explain briefly why together they prove a statement for every positive integer n. [2]

A2 For the claim $1 + 3 + 5 + \ldots + (2n - 1) = n^2$, verify the base case and write down the inductive hypothesis. [2]

SECTION B · Standard

B1 Complete the inductive step for $1 + 3 + 5 + \ldots + (2n - 1) = n^2$, and state the conclusion. [3]

B2 Prove by induction that the sum of the first n positive integers is $n(n + 1)/2$. [5]

B3 Prove by induction that $2 + 4 + 8 + \ldots + 2^n = 2^{n+1} - 2$. [5]

B4 A student proves the inductive step for a formula but skips the base case. Explain why nothing has been proved, using the false claim $1 + 3 + \ldots + (2n - 1) = n^2 + 1$ as an illustration. [3]

SECTION C · Stretch

C1 Prove by induction that the sum of $r \times 2^r$ from $r = 1$ to n equals $(n - 1)2^{n+1} + 2$ for all positive integers n. [6]