inkMaths

ANSWER BOOK

Quadratic functions

Algebra and functions · Year 12–13

7 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Half the 8, square it, correct for it: $(x + 4)^2 - 16 + 3 = (x + 4)^2 - 13$. The square is smallest when its bracket is zero, so the minimum point is $(-4, -13)$. M1 for halving and squaring the 8, A1 for the completed square, B1 for the minimum point. Reading the minimum as $(4, -13)$ is the sign slip to watch, since the bracket vanishes at $x = -p$. [3]

A2 $b^2 - 4ac = 9 - 40 = -31$, which is negative, so there are no real roots. M1 for the discriminant, A1 for the conclusion. The parabola opens upward and its minimum sits above the axis, so it never touches down. Quote the value and then state the conclusion; a bare number scores half. [2]

SECTION B · Standard

B1 Take the 3 out of the x terms first. $3(x^2 - 4x) + 5 = 3[(x - 2)^2 - 4] + 5$, and distributing gives $3(x - 2)^2 - 12 + 5 = 3(x - 2)^2 - 7$. The minimum point is $(2, -7)$. M1 for taking the 3 out, A1 for the bracket, A1 for -7, B1 for the minimum point. That correction term must be multiplied by the 3 on the way back out, and answers that leave it as $-4 + 5 = 1$ lose two marks in one step. [4]

B2 A repeated root means the discriminant is zero: $k^2 - 100 = 0$, so $k = 10$ or $k = -10$. M1 for setting the discriminant to zero, A1 for both values. Two different parabolas, each just touching the axis, one touching to the left of the origin and one to the right. Giving only $k = 10$ halves the mark. [2]

B3 Two distinct roots need $16 - 4k > 0$, so $k < 4$. But $k = 0$ has to be excluded as well, because the equation is then linear and has a single root rather than two. So $k < 4$ and $k \neq 0$. M1 for a positive discriminant, A1 for $k < 4$, B1 for excluding $k = 0$. The vanishing leading coefficient is the part most answers miss, and it is the mark that separates a 3 from a 2. [3]

B4 Since $9^x = (3^x)^2$, substitute $u = 3^x$ to get $u^2 - 10u + 9 = 0$, which factorises as $(u - 1)(u - 9) = 0$. Then $3^x = 1$ gives $x = 0$, and $3^x = 9$ gives $x = 2$. M1 for the substitution, A1 for the factorised quadratic, A1 for $x = 0$, A1 for $x = 2$. Spot the disguise, substitute, and return; stopping at $u = 1$ and $u = 9$ answers a question nobody asked. [4]

SECTION C · Stretch

C1 Set the discriminant to zero: $(4k)^2 - 4(k + 1)(9) = 0$, so $16k^2 - 36k - 36 = 0$. [5]
Dividing by 4 gives $4k^2 - 9k - 9 = 0$, which factorises as $(k - 3)(4k + 3) = 0$, so $k = 3$ or $k = -3/4$. M1 for setting the discriminant to zero, A1 for the quadratic in k, dM1 for factorising it, A1 for $k = 3$, A1 for $k = -3/4$. Both are legitimate: $k = 3$ turns the equation into $4x^2 + 12x + 9 = (2x + 3)^2 = 0$, and $k = -3/4$ into $(x - 6)^2/4 = 0$. The unknown sits in a and in b at once, so the discriminant becomes a quadratic in k, and discarding the fractional root because it looks untidy throws away the final mark. Only $k = -1$ would need excluding, and neither answer is -1.