inkMaths

ANSWER BOOK

Radians, arcs and small angles

Trigonometry · Year 12–13

7 questions · 21 marks

NAVY EDITION

SECTION A · Warm-up

A1 Since $180° = \pi$, dividing by 3 gives $60° = \pi/3$. Going the other way, $3\pi/4 = 3 \times 45° = 135°$. B1 B1 for the two conversions. Multiples of π over small denominators cover every angle the exact-value table knows. Exactly means the π stays; 1.047 is not an answer here. [2]

A2 The arc is $s = r\theta = 5 \times 0.8 = 4$ cm, and the area is $\frac{1}{2}r^2\theta = \frac{1}{2} \times 25 \times 0.8 = 10$ cm^2. B1 for the arc length, B1 for the area. No factor of 360 appears anywhere, which is the whole reward for working in radians. Both formulae are only valid with θ in radians, so an angle handed over in degrees must be converted first. [2]

SECTION B · Standard

B1 The arc is $9 \times 1.4 = 12.6$ cm, and the perimeter adds the two radii: $12.6 + 18 = 30.6$ cm. The area is $\frac{1}{2} \times 81 \times 1.4 = 56.7$ cm^2. B1 for the arc, B1 for adding the two radii, B1 for the area. Forgetting the two straight edges in the perimeter is the standard slip. [3]

B2 Rearranging $s = r\theta$ gives $\theta = 12/7.5 = 1.6$ radians, and then the area is $\frac{1}{2} \times 7.5^2 \times 1.6 = 45$ cm^2. M1 for rearranging $s = r\theta$, A1 for 1.6 radians, B1 for the area. The formulae run backwards as happily as forwards, and the θ that comes out is already in radians, so it drops straight into the area formula. Converting it to degrees on the way is wasted work and usually loses accuracy. [3]

B3 A segment is the sector with the triangle taken out. $\frac{1}{2} \times 36 \times 1.2 - \frac{1}{2} \times 36 \times \sin 1.2 = 21.6 - 16.78 = 4.82$ cm^2. The triangle uses $\frac{1}{2}r^2 \sin\theta$, with the two radii as its sides and θ between them. M1 for sector minus triangle, A1 for the two areas, A1 for 4.82 cm^2. Two things sink this question. The calculator must be in radian mode, since sin 1.2 in degrees is 0.0209 and the answer collapses; and the two terms are close together, so carry full accuracy into the subtraction rather than rounding each part first. [3]

SECTION C · Stretch

C1 $\cos 2\theta \approx 1 - (2\theta)^2/2 = 1 - 2\theta^2$, so the top is approximately $2\theta^2$. The bottom is $\theta \times \sin\theta \approx \theta \times \theta = \theta^2$. The ratio is $2\theta^2/\theta^2 = 2$, independent of θ. M1 for the expansion of $\cos 2\theta$, A1 for the numerator, M1 for $\sin\theta \approx \theta$ below, A1 for the value 2. The replacement for cos must use the angle actually present, 2θ, squared whole: $(2\theta)^2 = 4\theta^2$, not $2\theta^2$. [4]

C2 sin 0.1 = 0.0998…, within a fifth of a percent of 0.1. In radians a small angle's arc and sine are nearly the same length, which is what the approximation says. In degrees the claim reads $\sin 0.1^\circ \approx 0.1$, but $\sin 0.1^\circ = 0.0017\ldots$: the number 0.1 no longer measures the arc, so the geometry behind the approximation is gone. B1 for sin 0.1 = 0.0998, B1 for why arc and sine nearly agree, B1 for $\sin 0.1^\circ = 0.0017$, B1 for why the degree version fails. [4]