inkMaths

ANSWER BOOK

Recurrence relations

Further Pure 2 · Year FM

6 questions · 32 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Complementary function: $A4^n$. For the constant, try $u_n = c$, so $c = 4c - 3$ and $c = 1$. Then $u_n = A4^n + 1$, and $u_1 = 2$ gives $4A + 1 = 2$, so $A = 1/4$. Hence $\mathbf{u_n} = 4^{n-1} + 1$. B1 for the complementary function, M1 for a constant particular solution, A1 for $c = 1$, A1 for the final formula. The first four terms are 2, 5, 17, 65 either way. [4]

A2 The right-hand side is linear in n, so try a particular solution $an + b$, so $a(n + 1) + b = 2(an + b) + n$. Comparing coefficients of n gives $a = 2a + 1$, so $a = -1$; the constants give $a + b = 2b$, so $b = -1$. With the complementary function $A2^n$, $u_1 = 1$ gives $2A - 2 = 1$, so $A = 3/2$ and $\mathbf{u_n} = 3(2^{n-1}) - n - 1$. M1 for trying $an + b$, A1 for $a = -1$, A1 for $b = -1$, M1 for using the initial condition, A1 for the final formula. Checking $n = 3$: $12 - 4 = 8$, and the recurrence gives 8 too. [5]

SECTION B · Standard

B1 Auxiliary equation $m^2 - 5m + 6 = 0$, so $m = 2$ or $m = 3$ and $u_n = A2^n + B3^n$. The conditions give $2A + 3B = 1$ and $4A + 9B = 4$, so $B = 2/3$ and $A = -1/2$. Tidying, $\mathbf{u_n} = 2(3^{n-1}) - 2^{n-1}$. M1 for the auxiliary equation, A1 for the two roots, B1 for the general solution, M1 for using both conditions, A1 for A and B, A1 for the final formula. Checking $n = 3$: $18 - 4 = 14$, and $5(4) - 6(1) = 14$. [6]

B2 Auxiliary equation $m^2 - 6m + 9 = 0$ has the repeated root $m = 3$, so the general solution needs the extra factor of n and reads $u_n = (A + Bn)3^n$. The conditions give $3(A + B) = 3$ and $9(A + 2B) = 18$, so $A + B = 1$ and $A + 2B = 2$, giving $B = 1$ and $A = 0$. Hence $\mathbf{u_n} = n3^n$. M1 for the auxiliary equation, A1 for the repeated root, B1 for the general solution with its factor of n, M1 for the two conditions, A1 for A and B, A1 for the final formula. Checking $n = 3$: 81, and $6(18) - 9(3) = 81$. [6]

B3 Base case: $4^0 + 1 = 2$, as given. Assume $u_k = 4^{k-1} + 1$. Then $u_{k+1} = 4(4^{k-1} + 1) - 3 = 4^k + 4 - 3 = 4^k + 1$, which is the formula at $n = k + 1$. B1 for the base case, M1 for substituting the hypothesis into the recurrence, A1 for $4^k + 1$, A1 for the conclusion. True at $n = 1$ and inherited at each step, so true for all positive integers n. [4]

SECTION C · Stretch

C1 Auxiliary equation $m^2 + m - 6 = 0$ gives $m = 2$ or $m = -3$. For the constant 12, try $u_n = k$, giving $k + k - 6k = -4k = 12$, so $k = -3$. Then $u_n = A2^n + B(-3)^n - 3$, and the conditions give $2A - 3B = -1$ and $4A + 9B = 13$, so $A = 1$ and $B = 1$. Hence $\mathbf{u_n} = 2^n + (-3)^n - 3$. M1 for the auxiliary equation, A1 for the two roots, M1 for a constant particular solution, A1 for $k = -3$, M1 for the two conditions, A1 for A and B, A1 for the final formula. Checking $n = 3$: $8 - 27 - 3 = -22$, and $12 - 10 + 6(-4) = -22$. [7]