



QUESTION WORKBOOK

Reduction formulae

Further Pure 2 · Year FM

7 questions · 34 marks

PRINT EDITION

SECTION A · Warm-up

A1 Use integration by parts to evaluate $\int xe^x dx$ between 0 and 1. [3]

A2 $I_n = \int \sin^n x dx$ between 0 and $\pi/2$ satisfies $nI_n = (n-1)I_{n-2}$. Given $I_0 = \pi/2$ and $I_1 = 1$, find I_4 and I_5 . [4]

SECTION B · Standard

B1 $I_n = \int \sin^n x dx$ between 0 and $\pi/2$. Show that $nI_n = (n-1)I_{n-2}$. [5]

B2 $I_n = \int x^n e^x dx$ between 0 and 1. Show that $I_n = e - nI_{n-1}$, and hence find I_3 . [6]

B3 $I_n = \int \tan^n x dx$ between 0 and $\pi/4$. Show that $I_n = 1/(n-1) - I_{n-2}$ for $n \geq 2$, and find I_4 . [6]

B4 Explain why a reduction formula on its own is not an answer, and how you choose the starting value. [3]

SECTION C · Stretch

C1 $I_n = \int x^n e^{-x} dx$ from 0 to infinity. Show that $I_n = nI_{n-1}$ and hence evaluate I_5 . [7]