inkMaths

ANSWER BOOK

Route inspection

Decision Mathematics 1 · Year FM

6 questions · 24 marks

NAVY EDITION

SECTION A · Warm-up

A1 Every visit to a node uses one edge to arrive and one to leave, so a closed trail uses the edges at each node in pairs. A node of odd order would have an edge left over with no partner, so no such trail exists unless every node has even order. B1 for the edges at each node being used in pairs, B1 for the odd node leaving one unpaired. [2]

A2 AB with DE: 5 + 9 = **14**. AD with BE: 9 + 9 = **18**. AE with BD: 13 + 7 = **20**. B1 B1 B1 for the three pairings with their costs. Four odd nodes always give exactly three pairings, and the specification caps the number at four for that reason. List all three even when one is obviously worst, since the marks are for the comparison. [3]

SECTION B · Standard

B1 The cheapest pairing costs 14, so the route is 55 + 14 = **69**. The repeated stretches are **AB** and the shortest D to E path, which is **DC** and CE. M1 for choosing the cheapest pairing, A1 for 14, A1 for 69, B1 for the repeated edges. Every other edge is walked exactly once, and the route may start anywhere. Name the edges that are repeated as well as the pairing, since the final mark is for them. [4]

B2 An open route leaves two odd nodes unpaired, one at each end. To minimise the repetition, pair the two odd nodes with the cheapest connection and leave the others: pairing A with B costs 5, leaving D and E as the endpoints. The route is 55 + 5 = **60**, starting at **D** and finishing at **E**, or the other way round. M1 for leaving two odd nodes unpaired, A1 for the cheapest pairing of 5, A1 for 60, B1 for the two endpoints. [4]

B3 Two odd nodes may not be joined directly, and even when they are, a route through other nodes may be shorter. Repeating the edges of a shortest path raises the order of the two odd ends by one each and raises the intermediate nodes by two, leaving them even. Repeating an edge between two even nodes would make both odd, adding to the problem instead of solving it. B1 for a direct edge not always existing, B1 for a path through other nodes possibly being shorter, B1 for repeating an edge between even nodes making both odd. [3]

SECTION C · Stretch

C1 Degrees: A 3, B 3, C 5, D 4, E 4, F 3. The odd vertices are **A,** B, C and F, and the edges total **68**. [8]
Shortest paths between them: AB = 7 along the edge; AC = 5; BC = 4; AF = 13 by A, C, E, F; BF = 12 by B, C, E, F; and CF = **8** by C, E, F, which beats the direct edge CF of 10.
Pairings: AB + CF = 7 + 8 = **15**; AC + BF = 5 + 12 = **17**; AF + BC = 13 + 4 = **17**.
The cheapest is 15, so the shortest closed route has length 68 + 15 = **83**, and the repeated edges are **AB,** CE and EF.
B1 for the four odd vertices, B1 for the total of 68, M1 for the shortest paths between odd vertices, A1 for the C to F path of 8, M1 for the three pairings, A1 for 15, A1 for 83, A1 for the repeated edges. The trap is the pair C and F. Taking the direct edge of 10 gives a pairing of 17 and a route of 85, and the two marks for the shortest paths are lost with it. Check every connection between odd vertices for a cheaper way round before pairing.