inkMaths

ANSWER BOOK

Simple harmonic motion

Further Mechanics 2 · Year FM

6 questions · 28 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $\ddot{x} = -\omega^2 x$. The acceleration is proportional to the displacement from a fixed point and directed back towards it. The speed is greatest at the centre, where $x = 0$ and the acceleration vanishes. The acceleration is greatest at the ends, where the particle is momentarily at rest. B1 for the defining equation, B1 for greatest speed at the centre, B1 for greatest acceleration at the ends. The minus sign is part of the definition and dropping it loses the mark. [3]

A2 Maximum speed = $a\omega$ = **2** m/s. Maximum acceleration = $a\omega^2 = 0.4 \times 25 =$ **10** m/s². Period = $2\pi/\omega$ = **1.26** s. B1 B1 B1, one for each of the three values. [3]

SECTION B · Standard

B1 Using $v^2 = \omega^2(a^2 - x^2)$ twice: $2.56 = \omega^2(a^2 - 0.09)$ and $1.44 = \omega^2(a^2 - 0.16)$. Dividing removes ω^2: $2.56/1.44 = (a^2 - 0.09)/(a^2 - 0.16)$, so $1.778(a^2 - 0.16) = a^2 - 0.09$ and $0.778a^2 = 0.194$, giving $a^2 = 0.25$ and $a =$ **0.5** m. Then $\omega^2 = 2.56/0.16 = 16$, so $\omega =$ **4** rad/s. M1 for using $v^2 = \omega^2(a^2 - x^2)$, A1 A1 for the two equations, M1 for eliminating ω^2, A1 for $a = 0.5$ m, A1 for $\omega = 4$ rad/s. [6]

B2 Timing from the centre, $x = 0.5 \sin 4t$. Setting $0.25 = 0.5 \sin 4t$ gives $\sin 4t = 0.5$, so $4t = \pi/6$ and $t = \pi/24 =$ **0.131** s. The particle returns to the same displacement when $4t = 5\pi/6$, that is $t = 5\pi/24 =$ **0.654** s, on the way back. M1 for $x = 0.5 \sin 4t$, M1 for solving $\sin 4t = 0.5$, A1 for 0.131 s, M1 for $4t = 5\pi/6$, A1 for 0.654 s. [5]

B3 A larger amplitude means a longer distance to travel. It also means a proportionally larger restoring force and so a proportionally larger speed at every corresponding point. Because the force is proportional to the displacement, the two effects cancel exactly and $T = 2\pi/\omega$ whatever the amplitude. B1 for the greater distance, B1 for the proportionally greater restoring force and speed, B1 for the two effects cancelling. That is what makes a pendulum useful as a clock. [3]

SECTION C · Stretch

C1 Write the acceleration as $v\,dv/dx$, so $v\,dv/dx = -\omega^2 x$. Separating gives $\int v\,dv = -\omega^2\int x\,dx$, so $\frac{1}{2}v^2 = -\frac{1}{2}\omega^2x^2 + c$. At the extreme of the motion $x = a$ and $v = 0$, so $c = \frac{1}{2}\omega^2a^2$. Substituting back and doubling gives $\mathbf{v^2} = \omega^2(a^2 - x^2)$. The result contains no time, so it answers position and speed questions in one step. [8]

For the second part, time from the centre with $x = a \sin \omega t$. Then $|x| \le a/2$ requires $|\sin \omega t| \le \frac{1}{2}$, that is ωt between 0 and $\pi/6$, or between $5\pi/6$ and $7\pi/6$, or between $11\pi/6$ and 2π. Those stretches measure $\pi/6$, $\pi/3$ and $\pi/6$, totalling $\mathbf{2\pi/3}$ out of a full 2π, so the proportion is **1/3**.

M1 for writing the acceleration as $v\,dv/dx$, M1 for separating and integrating, A1 for $\frac{1}{2}v^2 = -\frac{1}{2}\omega^2x^2 + c$, A1 for evaluating c and reaching the printed result, M1 for $|\sin \omega t| \le \frac{1}{2}$, A1 for the three intervals, A1 for the total $2\pi/3$, A1 for the proportion 1/3. The particle covers the middle half of its range in a third of the time, so it is far quicker through the centre than the ends. The answer depends on neither a nor ω, and choosing $x = a \cos \omega t$ instead gives the same third with the intervals shifted.