inkMaths

ANSWER BOOK

Simultaneous equations and inequalities

Algebra and functions · Year 12–13

8 questions · 33 marks

NAVY EDITION

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SECTION A · Warm-up

A1 Adding the two equations eliminates y, giving $3x = 9$, so $x = 3$, and substituting back into either equation gives $y = 1$. M1 for a correct elimination or substitution, A1 for x, A1 for y. Putting (3, 1) into both original equations is the one-line check the answer deserves, and it costs less time than losing the accuracy marks. [3]

A2 Factorise to $(x - 4)(x + 3) < 0$, so the boundary values are $x = -3$ and $x = 4$. The parabola opens upwards, so it lies below the axis strictly between its roots, giving $-3 < x < 4$. M1 for factorising, A1 for the roots, A1 for the inequality. A sketch is the argument here. Quote the roots, then read the sign off the shape, and never simply write the roots down joined by inequality signs without checking which way round they go. [3]

SECTION B · Standard

B1 Equating the two expressions gives $x^2 - 3x + 4 = x + 1$, so $x^2 - 4x + 3 = 0$, which factorises as $(x - 1)(x - 3) = 0$. Then $x = 1$ gives $y = 2$ and $x = 3$ gives $y = 4$, so the points are (1, 2) and (3, 4). M1 for equating, A1 for the three-term quadratic, M1 for solving, A1 for the x values, A1 for the y values. The answers come in matched pairs, and pairing them the wrong way round costs the final mark even when both lists are right. [5]

B2 Substituting gives $x^2 = 2x + c$, so $x^2 - 2x - c = 0$. Tangency means the line meets the curve exactly once, so this quadratic has a repeated root and its discriminant is zero: $b^2 - 4ac = 4 + 4c = 0$, giving $c = -1$. M1 for substituting, A1 for the quadratic, M1 for setting the discriminant to zero, A1 for c. The discriminant of the substituted quadratic is geometry in disguise. Positive means two crossings, zero means a tangent, negative means the line misses the curve entirely. [4]

B3 Multiplying by x is unsafe, because the inequality flips whenever x is negative. Multiply by x^2 instead, which is positive for every $x \neq 0$: $2x > x^2$, so $x^2 - 2x < 0$ and $x(x - 2) < 0$, giving $0 < x < 2$. M1 for a valid method that handles the sign, A1 for the quadratic inequality, A1 for the interval, B1 for checking. Test three values: $x = 1$ gives $2 > 1$, true; $x = 3$ gives $2/3 > 1$, false; $x = -1$ gives $-2 > 1$, false. Multiplying naively by x produces $x < 2$ and wrongly hands the whole negative axis to the solution set. [4]

B4 Factorise to $(x + 5)(x - 3) \geq 0$, with boundary values -5 and 3. The parabola opens upwards, so it lies on or above the axis outside its roots, giving $x \leq -5$ or $x \geq 3$. In set notation that is $\{x : x \leq -5\} \cup \{x : x \geq 3\}$. M1 for factorising, A1 for the two regions, A1 for correct set notation. The two pieces need the union symbol rather than a comma, since they are separate stretches of the number line with a gap between them, and writing $3 \leq x \leq -5$ describes nothing at all. [3]

SECTION C · Stretch

C1 Substitute the linear equation into the quadratic one: $x^2 + (2x - 5)^2 = 25$, so $x^2 + 4x^2 - 20x + 25 = 25$, which reduces to $5x^2 - 20x = 0$. Factorising, $5x(x - 4) = 0$, so $x = 0$ or $x = 4$. Then $y = 2(0) - 5 = -5$ and $y = 2(4) - 5 = 3$, giving the points $(0, -5)$ and $(4, 3)$. M1 for substituting, A1 for the quadratic, M1 for solving, A1 for the x values, A1 for the y values. Substitute the line into the circle rather than the other way round, since making y the subject of $x^2 + y^2 = 25$ drags a square root through the whole calculation for no benefit. [5]

C2 Setting the two equal gives $x^2 + x + 1 = kx - 3$, so $x^2 + (1 - k)x + 4 = 0$. No intersection means this quadratic has no real roots, so its discriminant is negative: $(1 - k)^2 - 16 < 0$, that is $(1 - k)^2 < 16$. Taking square roots carefully, $-4 < 1 - k < 4$, so $-3 < k < 5$. M1 for equating, A1 for the quadratic in x, M1 for the discriminant condition, A1 for $(1 - k)^2 < 16$, M1 for solving, A1 for the interval. Two traps sit here. The discriminant involves the coefficients of the substituted quadratic, not of the original curve, and $(1 - k)^2 < 16$ gives an interval rather than $1 - k < 4$ alone. Check with $k = 0$: the line $y = -3$ and the curve whose minimum is 0.75 clearly never meet, and 0 does lie in the answer. [6]