



---

QUESTION WORKBOOK

# Subgroups, Lagrange's theorem and isomorphism

Further Pure 2 · Year FM

7 questions · 27 marks

PRINT EDITION

**SECTION A** · Warm-up

- A1** State Lagrange's theorem and one immediate consequence for the order of an element. [2]

---

---

- A2** A cyclic group of order 6 is generated by  $g$ . List its subgroups and check them against Lagrange's theorem. [3]

**SECTION B** · Standard

- B1** A group  $G$  has order 8. State the possible orders of its subgroups, and prove that  $G$  cannot contain an element of order 3. [4]

- B2** Prove that a group of prime order  $p$  is cyclic and has no proper subgroups other than the identity. [4]

- B3** Decide whether the cyclic group of order 4 and the group  $\{1, 3, 5, 7\}$  under multiplication modulo 8 are isomorphic, with a reason. [3]

- B4** The symmetries of a square form a group of order 8. Find a subgroup of order 4 in two genuinely different ways, and one of order 2. [4]

## SECTION C · Stretch

- C1** The group  $D$  of the symmetries of a square has order 8. The set  $U = \{1, 3, 5, 7, 9, 11, 13, 15\}$  forms a group of order 8 under multiplication modulo 16. Determine, with full justification, whether  $D$  and  $U$  are isomorphic. [7]