InkMaths

ANSWER BOOK

Successive impacts and impacts with a wall

Further Mechanics 1 · Year FM

7 questions · 29 marks

NAVY EDITION

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SECTION A · Warm-up

A1 [2]

The floor is attached to the Earth, so it is an external body and the ball on its own is not an isolated system: the impulse it receives changes its momentum. What still applies is the law of restitution, which says the rebound speed is e times the approach speed. B1 for the floor being an external body, B1 for restitution still applying.

A2 [3]

If the drop height is h, the arrival speed is $\sqrt{(2gh)}$, the rebound speed is $e\sqrt{(2gh)}$, and the height reached is e^2h. So the rebound reaches $0.49 \times 3 =$ **1.47** m. M1 for a rebound height of e^2h, A1 for 0.49×3, A1 for 1.47 m. The value of g never has to be worked out.

SECTION B · Standard

B1 [4]

The ball arrives at $\sqrt{(2gh)}$ and leaves at $e\sqrt{(2gh)}$, so it rises to e^2h. Repeating gives heights $h, e^2h, e^4h, \ldots$, a geometric sequence of ratio e^2.

The first height is travelled once, downwards. Every later height is travelled twice, up and down.

Total $= h + 2(e^2h + e^4h + \ldots) = h + 2e^2h/(1 - e^2)$, the sum to infinity being valid because $e^2 < 1$.

Over a common denominator: $h(1 - e^2 + 2e^2)/(1 - e^2) =$ **$h(1 + e^2)/(1 - e^2)$** as required. M1 for the heights forming a geometric sequence of ratio e^2, A1 for the sum to infinity, M1 for doubling all but the first height, A1 for the printed result. The factor of 2 on all but the first term is where this proof is usually lost.

B2 [4]

The heights form a geometric sequence 3, 1.47, 0.7203, … with ratio $e^2 = 0.49$.

The first drop is travelled once and every later height twice, so the total is $3 + 2(1.47 + 0.7203 + \ldots) = 3 + 2(1.47/0.51) = 3 + 5.765 =$ **8.76** m.

The closed form $h(1 + e^2)/(1 - e^2) = 3(1.49)/0.51$ gives the same. M1 for the sequence of ratio 0.49, M1 for doubling all but the first drop, A1 for 1.47/0.51, A1 for 8.76 m. Doubling the first drop as well is the usual slip and gives 11.76.

B3 [5]

First impact. Momentum: $2(6) = 2v_A + v_B$. Restitution: $v_B - v_A = 0.5(6 - 0) = 3$. Substituting the second into the first: $3v_A + 3 = 12$, so $v_A =$ **3** m/s and $v_B =$ **6** m/s. Since $3 < 6$, A does not pass through B, so the answer is physically possible.

At the wall. B approaches at 6 and rebounds at $6 \times 1/3 =$ **2** m/s, now travelling back towards A.

After the wall. A is still moving forward at 3 m/s and B is moving backwards at 2 m/s, so they approach each other at **5** m/s and must meet again. M1 for conservation of momentum, M1 for the restitution equation, A1 for 3 m/s and 6 m/s, B1 for B rebounding at 2 m/s, A1 for the approach speed of 5 m/s and the conclusion.

B4 The rebound speed satisfies $v^2 = 2(9.8)(0.9) = 17.64$, so $v =$ **4.2** m/s. Then $e = 4.2/7 =$ **0.6**. M1 for $v^2 = 2gh$, A1 for $v = 4.2$ m/s, A1 for $e = 0.6$. [3]

SECTION C · Stretch

C1 **A** strikes B. Momentum: $3mu = 3mv_A + mv_B$, so $3v_A + v_B = 3u$. Restitution: $v_B - v_A = u/2$. [8]

Substituting: $3v_A + v_A + u/2 = 3u$, so $4v_A = 5u/2$ and $v_A =$ **5u/8**. Then $v_B = 5u/8 + 4u/8 =$ **9u/8**. Check $5u/8 < 9u/8$, so A does not overtake B.

B strikes C. B arrives at 9u/8 and C is at rest. Momentum: $m(9u/8) = mv_B' + 2mv_C$. Restitution: $v_C - v_B' = (1/2)(9u/8) = 9u/16$.

Substituting: $v_B' + 2v_B' + 9u/8 = 9u/8$, so $3v_B' = 0$ and $v_B' =$ **0**. B is brought to rest, and $v_C =$ **9u/16**.

Check: momentum 9mu/8 before, $0 + 2m(9u/16) = 9mu/8$ after.

Conclusion. A is still moving forward at 5u/8 and B is now at rest, so A closes on B at 5u/8 and **does** strike B again. C moves away at 9u/16, so the question is whether B can catch it. Work that third impact through and B leaves it at 45u/64 while C is still travelling at $9u/16 = 36u/64$, so **B** does catch C and a fourth impact follows. Judging it by B's speed while B is still at rest is the trap: the line above has already said B is about to be struck.

M1 for momentum at the first impact, M1 for restitution there, A1 for 5u/8, A1 for 9u/8, M1 for momentum and restitution at the second impact, A1 for B brought to rest, A1 for 9u/16, A1 for the conclusion that A strikes B again. Use the restitution equation for the impact in front of it, never for the pair A and C, which never touch. Feeding u rather than 9u/8 into the second restitution equation is the error that ends this question.