inkMaths

ANSWER BOOK

Summing series and the method of differences

Further algebra and series · Year FM

6 questions · 24 marks

NAVY EDITION

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SECTION A · Warm-up

A1 $\Sigma r = n(n + 1)/2$ and $\Sigma r^2 = n(n + 1)(2n + 1)/6$, both from $r = 1$ to n. B1 B1 for the two standard results. Everything polynomial in this topic reduces to these and the cubes formula. [2]

A2 $\Sigma r^3 = n^2(n + 1)^2/4$, so multiplying by 4 clears the fraction: $\Sigma 4r^3 = n^2(n + 1)^2$. At $n = 3$: $4(1 + 8 + 27) = 144$ and $9 \times 16 = 144$. Agreed. M1 for quoting $\Sigma r^3 = n^2(n + 1)^2/4$, A1 for the printed result, B1 for the check at $n = 3$. [3]

SECTION B · Standard

B1 Split: $\Sigma r^2 + 2\Sigma r = n(n + 1)(2n + 1)/6 + n(n + 1)$. Take out $n(n + 1)/6$: the bracket is $(2n + 1) + 6 = 2n + 7$, giving $n(n + 1)(2n + 7)/6$. Check at $n = 3$: $3 + 8 + 15 = 26$, and $3 \times 4 \times 13/6 = 26$. M1 for splitting into $\Sigma r^2 + 2\Sigma r$, A1 for both standard results, M1 for taking out $n(n + 1)/6$, A1 for the printed result. [4]

B2 The telescope cancels every inner term: the sum is $\frac{1}{2}(1 - 1/(2n + 1)) = n/(2n + 1)$. Check at $n = 3$: $1/3 + 1/15 + 1/35 = 3/7$, and $n/(2n + 1) = 3/7$. M1 for writing out enough terms, A1 for the surviving pair, A1 for $\frac{1}{2}(1 - 1/(2n + 1))$, A1 for $n/(2n + 1)$. Only the first positive and last negative fractions survive. [4]

B3 Subtract two full sums: Σ to $2n$ minus Σ to n = $2n(2n + 1)(4n + 1)/6 - n(n + 1)(2n + 1)/6 = n(2n + 1)(7n + 1)/6$, after taking out $n(2n + 1)/6$. At $n = 2$: $9 + 16 = 25$ and $2 \times 5 \times 15/6 = 25$. M1 for the difference of two sums, A1 for both standard results, M1 for taking out $n(2n + 1)/6$, A1 for $n(2n + 1)(7n + 1)/6$. Sums that start above 1 are always a difference of two sums that start at 1. [4]

SECTION C · Stretch

C1 $1/(r(r+2)) = A/r + B/(r+2)$ gives $1 = A(r+2) + Br$. Putting $r = 0$ gives $A = ½$ and $r = -2$ gives $B = -½$, so **$1/(r(r + 2))$** $= ½(1/r - 1/(r + 2))$. [7]

Now write the terms out. Each negative part cancels the positive part **two** rows below, not one, so two terms survive at each end rather than one:

$2\Sigma = (1/1 - 1/3) + (1/2 - 1/4) + (1/3 - 1/5) + \dots + (1/(n-1) - 1/(n+1)) + (1/n - 1/(n+2))$.

The survivors are **1** and $1/2$ at the top and **$-1/(n$** $+ 1)$ and $-1/(n + 2)$ at the bottom.

So $\Sigma = ½[3/2 - 1/(n + 1) - 1/(n + 2)]$. Combining the last two over a common denominator: $1/(n + 1) + 1/(n + 2) = (2n + 3)/((n + 1)(n + 2))$.

Hence $\Sigma =$ **$3/4$** $- (2n + 3)/(2(n + 1)(n + 2))$, as required. Check at $n = 2$: $3/4 - 7/24 = 11/24$, and $1/3 + 1/8 = 11/24$.

As $n \to \infty$ the subtracted fraction tends to 0, so the sum to infinity is **$3/4$**.

M1 for setting up the partial fractions, A1 for the printed pair, M1 for writing out enough terms, A1 for the four survivors, A1 for $½[3/2 - 1/(n + 1) - 1/(n + 2)]$, A1 for the printed result, B1 for the sum to infinity. Writing out enough terms to see both ends of the cancellation is the method mark. Assuming one survivor at each end, as in the gap-one case, gives $1/2$ in place of $3/4$ and loses everything after it.