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ANSWER BOOK

Tangents, normals and loci of conics

Further Pure 1 · Year FM

6 questions · 23 marks

NAVY EDITION

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SECTION A · Warm-up

A1 The tangent is $ty = x + at^2$, with gradient $1/t$. The normal is perpendicular, so its gradient is $-t$. B1 for the tangent, B1 for the normal gradient. [2]

A2 Here $4a = 16$ so $a = 4$, and the point is $(at^2, 2at) = (16, 16)$. The tangent $ty = x + at^2$ becomes $2y = x + 16$, that is $y = x/2 + 8$. Checking: $16^2 = 256 = 16 \times 16$, so the point really is on the curve. B1 for $a = 4$, M1 for substituting into $ty = x + at^2$, A1 for $2y = x + 16$. [3]

SECTION B · Standard

B1 Substituting: $(2x + 2)^2 = 16x$ gives $4x^2 - 8x + 4 = 0$, that is $4(x - 1)^2 = 0$. The repeated root $x = 1$ means the line touches rather than crosses, at the point $(1, 4)$. M1 for substituting, A1 for $4x^2 - 8x + 4 = 0$, A1 for the repeated root, A1 for $(1, 4)$. The condition $c = a/m$ agrees: $a = 4$ and $m = 2$ give $c = 2$. [4]

B2 Differentiating implicitly: $y + x\, dy/dx = 0$, so $dy/dx = -y/x = -1$ at $(4, 4)$. The tangent is $y - 4 = -(x - 4)$, that is $y = 8 - x$. The normal has gradient 1: $y = x$, which is the hyperbola's own axis of symmetry through that point. M1 for differentiating implicitly, A1 for $dy/dx = -1$, A1 for $y = 8 - x$, A1 for $y = x$. [4]

B3 A chord $y = 4x + c$ meets the curve where $y^2 = 4(y - c)$, that is $y^2 - 4y + 4c = 0$. The two y-values always sum to 4 regardless of c, so the midpoint has $y = 2$ every time. The locus is the line $y = 2$, taken inside the parabola. M1 for substituting $y = 4x + c$, A1 for $y^2 - 4y + 4c = 0$, M1 for the sum of the roots, A1 for $y = 2$. [4]

SECTION C · Stretch

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C1 The tangent at P is $ty = x + at^2$, of gradient $1/t$, so the normal has gradient **$-t$**. [6]
Its equation is $y - 2at = -t(x - at^2)$, that is **y** $+ tx = 2at + at^3$.
Q lies on it, so $2aT + atT^2 = 2at + at^3$. Divide by a, which is never zero:
$2T + tT^2 = 2t + t^3$.
Collect everything on one side and factorise, expecting $T = t$ to be a root, since P itself is on both the normal and the curve:
$t(T^2 - t^2) + 2(T - t) = 0$, so $(T - t)[t(T + t) + 2] = 0$.
Q is not P, so $T \neq t$ and the second bracket must vanish: $t(T + t) = -2$, giving $T + t = -2/t$ and **T** $= -t - 2/t$ as required. Spotting the factor $(T - t)$ is what turns a cubic into one line; expanding and solving from scratch is where this question is lost.
With $a = 4$ and $t = 1$: $T = -1 - 2 =$ **-3**, so $Q = (aT^2, 2aT) =$ **(36,** $-24)$. Check: $(-24)^2 = 576 = 16 \times 36$, and the normal $y + x = 12$ does pass through $(36, -24)$. M1 for the normal gradient, A1 for its equation, M1 for substituting Q, M1 for factorising out $(T - t)$, A1 for the printed result, A1 for $(36, -24)$.

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